Probabilistic Reasoning for the Verification of Side-Channel Countermeasures

EGRAPHS talk

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TU WIEN Informatics

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I am not

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- A term rewriting expert

Side-Channel Attack

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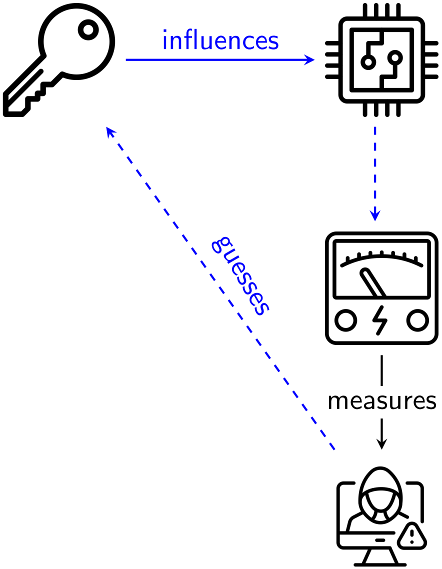
(b) Away from home

Power Side-Channel Attack



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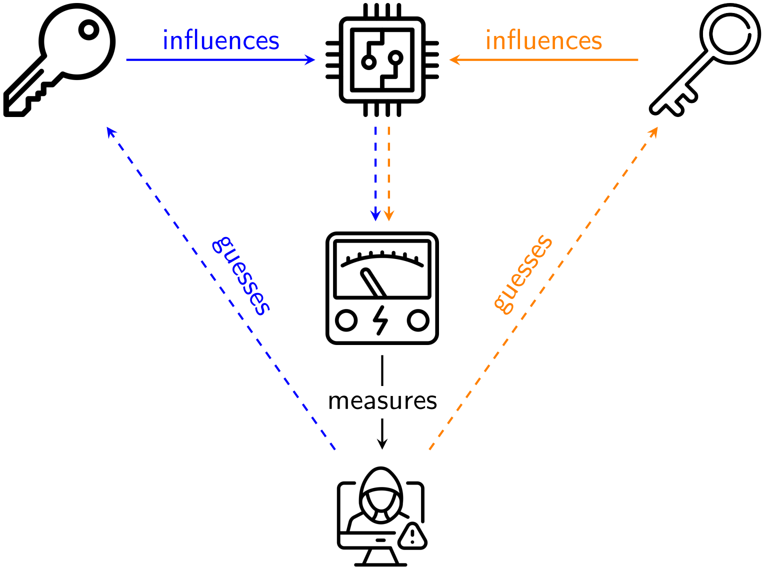

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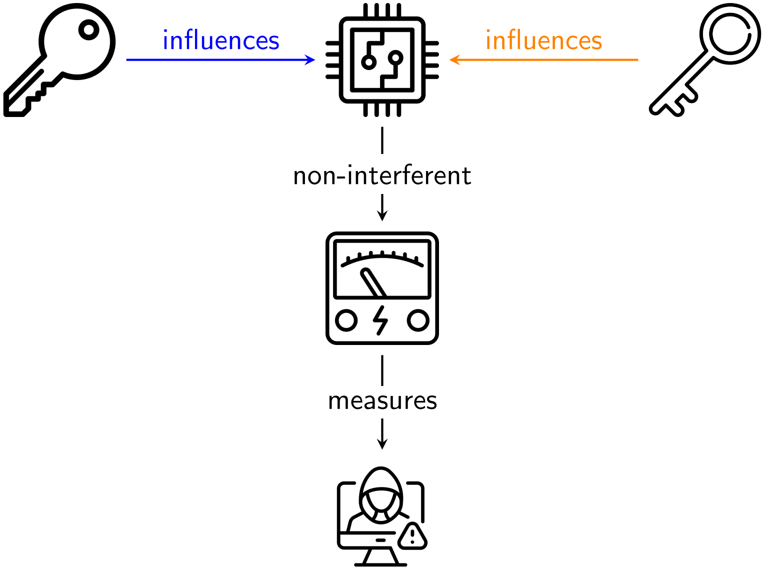

Shares for Secret Masking

Secret Masking

Let s be a secret, we split the secret into n shares $s_1, s_2, \ldots, s_n$ such that:

$$s = s_1 \oplus s_2 \oplus \ldots \oplus s_n$$

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$\mathbf{s}$	$\mathbf{s_1}$	$\mu\mathbf{P}$
0	0	$0W$
1	1	$1W$

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$\mathbf{s}$	$\mathbf{s_1}$	$\mathbf{s_2}$	$\mu\mathbf{P}$	$\mu^2\mathbf{P}$
0	0	0	$1W$	$1W^2$
0	1	1		
1	0	1	$1W$	$0W^2$
1	1	0		

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$\mathbf{s}$	$\mathbf{s_1}$	$\mathbf{s_2}$	$\mathbf{s_3}$	$\mu\mathbf{P}$	$\mu^{\mathbf{2}}\mathbf{P}$	$\mu^{\mathbf{3}}\mathbf{P}$
0	0	0	0	1.5W	0.75W^2	-0.75W^3
0	1	1	0			
0	1	0	1			
0	0	1	1			
1	0	0	1	1.5W	0.75W^2	0.75W^3
1	1	1	1			
1	0	1	0			
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Probe-Isolating Non-Interference

Abstract Goal

$$\forall s, s'.\ P(power|s) \sim P(power|s')$$

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Practical Goal (simplified)

$$\bigwedge_{W \in \mathcal{C}} \forall w, s_1, \ldots, s_n.\ probe(W) \Rightarrow \bigvee_{i=1}^{n} P(W = w | S_i = s_i) = P(W = w)$$

Proof by Refutation

Goal

Let $\mathcal{C}(\bar{x})$ be the definition of the circuit, and $\mathcal{P}(\bar{x})$ be the security property for some input $\bar{x}$. We wish to prove that the following is always true:

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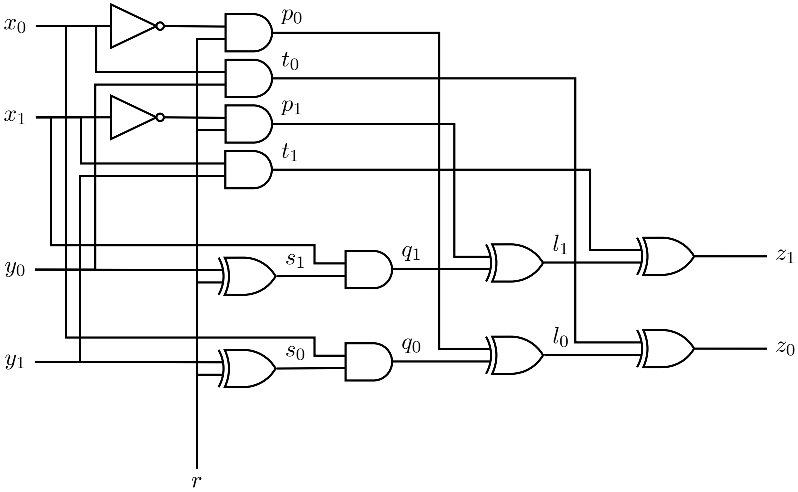
Proof by Refutation

We prove the negation is unsatisfiable on $\bar{a}$:

$$\mathcal{C}(\bar{a}) \wedge \neg \mathcal{P}(\bar{a})$$

We eliminated the universal quantifier and have a ground formula in CNF.

PINI AND Gate Example [Sal23]



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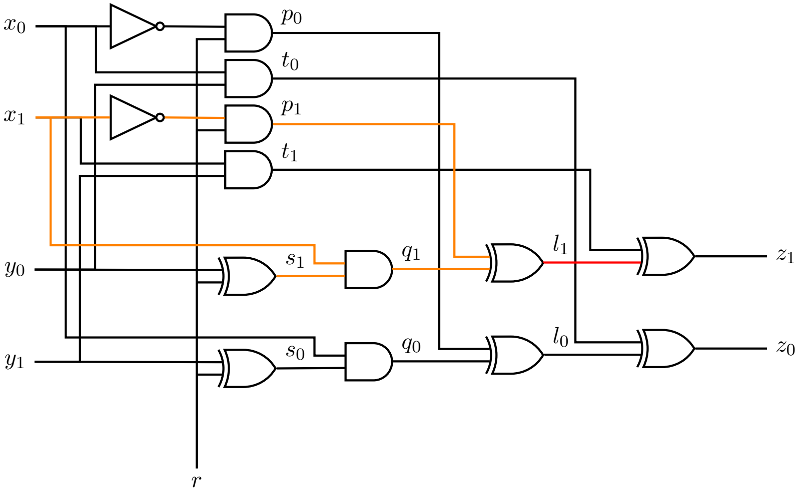
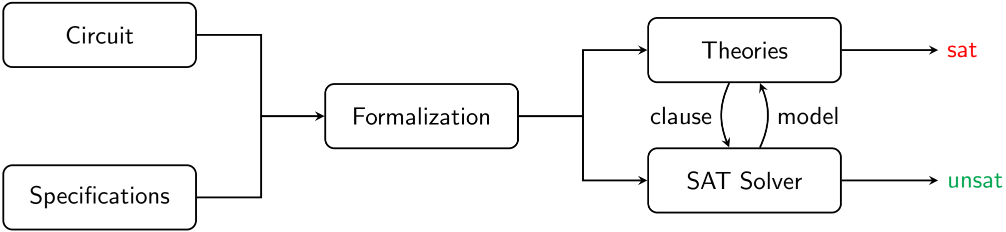


SMT-like Verification for PINI



SAT and Theory split

Objective

$$\bigwedge_{W \in \mathcal{C}} probe(W) \Rightarrow \bigvee_{i=1}^{n} P(W = w | S_i = s_i) = P(W = w)$$

SAT Part

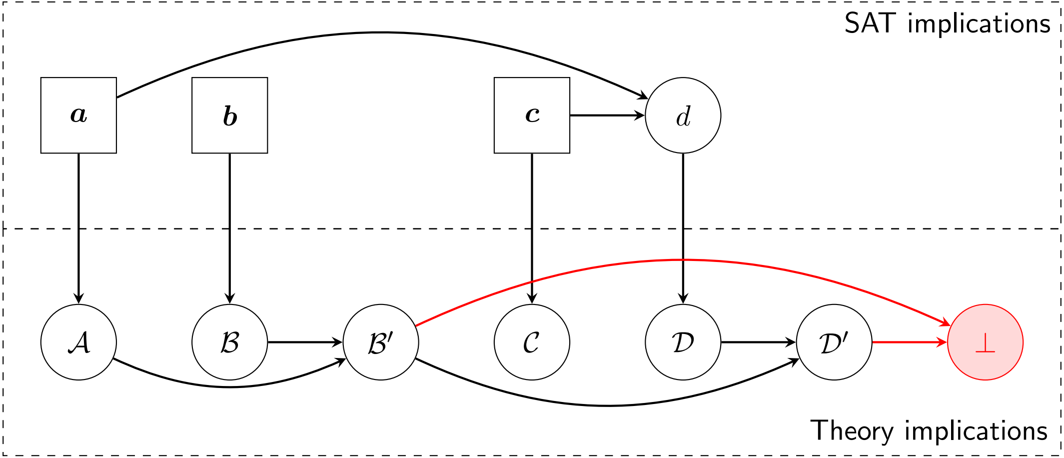
$$\bigwedge_{W \in \mathcal{C}} p_W \Rightarrow \bigvee_{i=1}^{n} b_i$$

Theory Part

$$p_W \Leftrightarrow probe(W)$$

$$b_i \Leftrightarrow P(W = w | S_i = s_i) = P(W = w)$$

SMT: Theory Conflict Analysis



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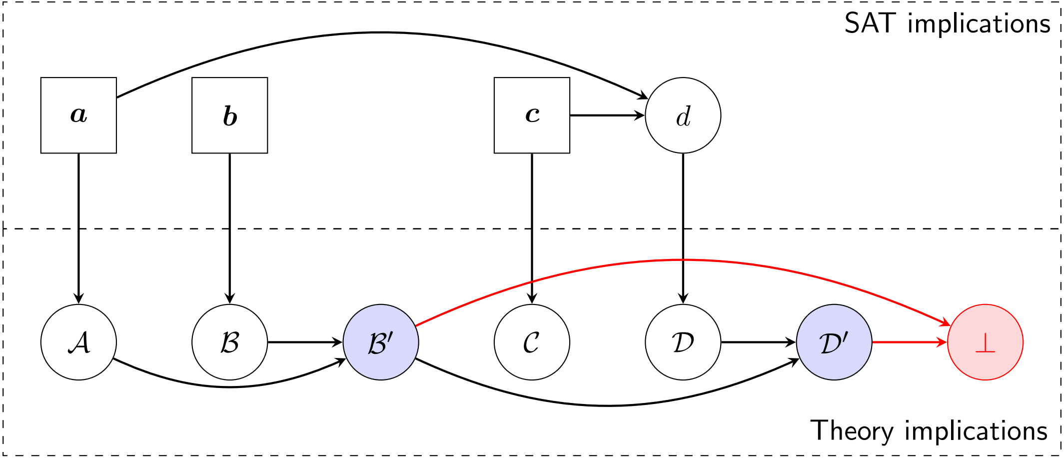

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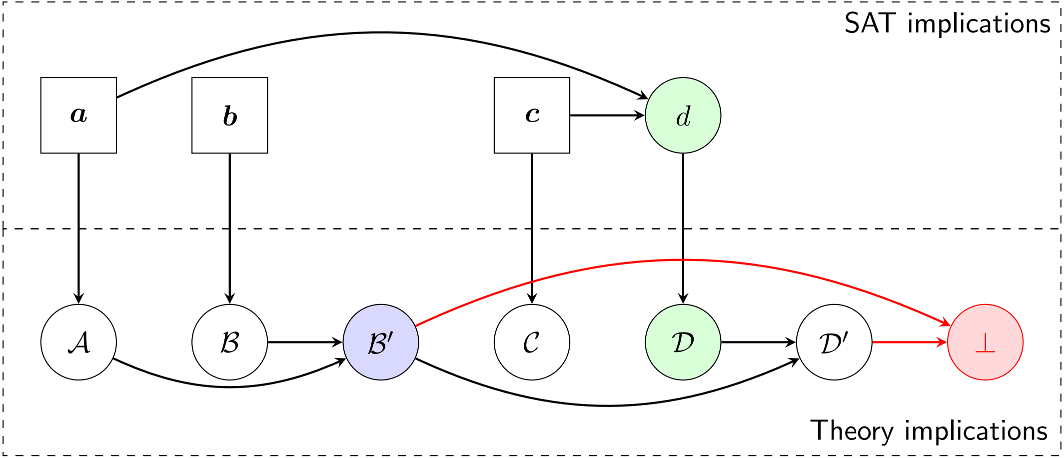

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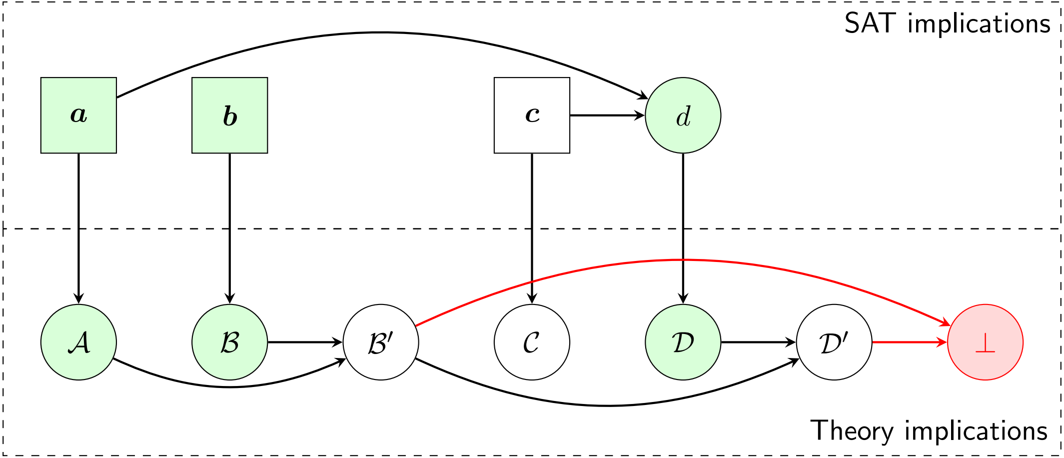
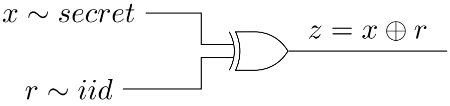


Motivating Example



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Prove that the output z is independent of the secret x.

$$P(z|x,r) = x + r - 2x * r \qquad \text{(circuit)}$$

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What do we need?

- Probability reasoning
- Equality reasoning
- Non-linear real arithmetic

Solving Steps

$P(z\|x,r) = x + r - 2x * r$	input
$P(z,x) \neq P(z)P(x)$	input
$P(z,x) = P(z\|x)P(x)$	chain rule on $P(z,x)$
$P(z\|x) = P(z\|x,r)P(r\|x) + P(z\|x,1-r)P(1-r\|x)$	law of total probability
$P(r\|x) = P(1-r\|x) = P(r) = 0.5$	r is uniform iid
$P(z\|x) = 0.5(x + r - 2x * r) + 0.5(x + (1-r) - 2x * (1-r))$	substitution
$P(z\|x) = 0.5$	simplification
$P(z\|x) = P(z)$	independence
$P(z,x) = P(z)P(x)$	Conflict with input

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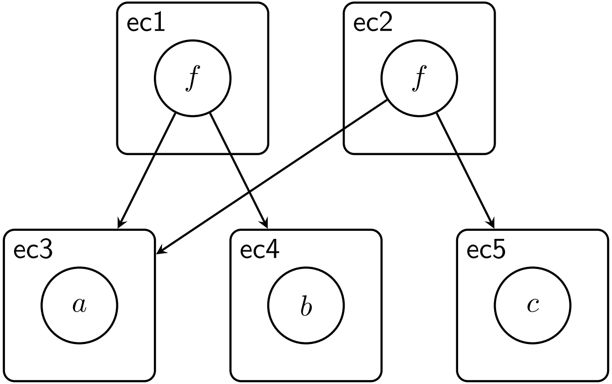
Egraphs



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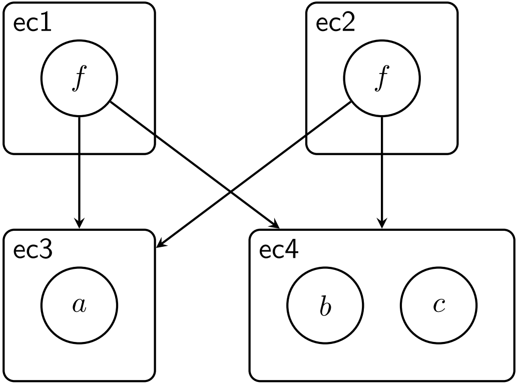

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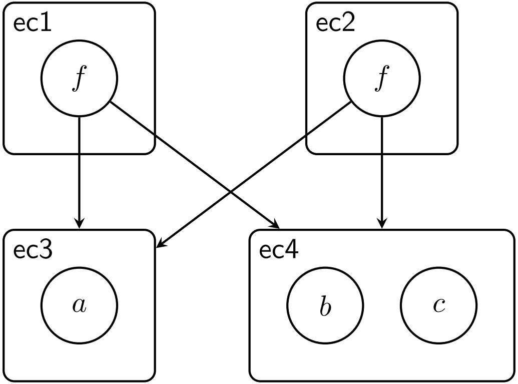

Congruence : $f(ec3, ec4)$

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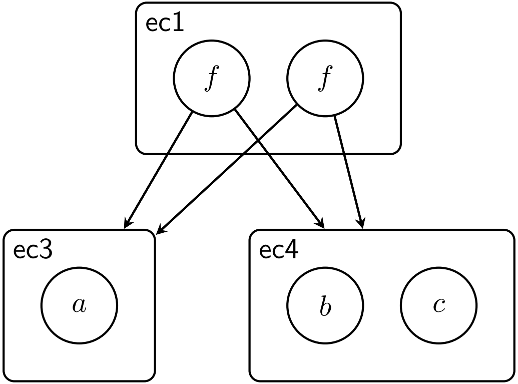

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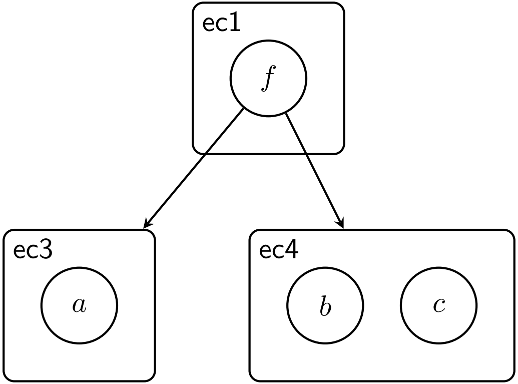

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- Compact set of equalities.
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Challenges

- Probability calculus is not well understood.
- Combination of theories.
- Polynomials are known to be difficult.
- Complex term introduction.

What about EGG? [WWF+20]

Similarities.

- Saturation of Egraphs.
- Rewrite rules for theory reasoning.

Differences.

- Conflict analysis and backtracking (known from SMT but more complicated).
- Proof production (known from SMT).
- Lemma generation (application specific).
- New term and function symbol introduction (technically not difficult with custom data structures).

Challenge I: Non-Linear Real Arithmetic

NRA is non terminating with rewrites [BN98]

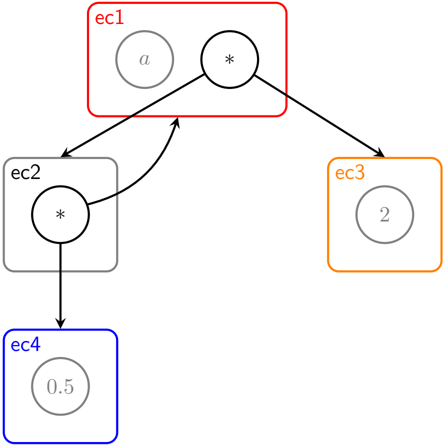
Terms like $(x * a) * \frac{1}{x}$ can be rewritten forever using associativity and commutativity of $*$ and the rule $x * \frac{1}{x} \rightarrow 1$.

Example in PINI

$P(z) = P(x) + P(r) - 2 * P(x) * P(r)$ becomes $P(z) = P(x) + P(r) - 2 * P(x) * 0.5$

Challenge I: Illustration of Redundant Cycles

$$(x * y) * z \longrightarrow x * (y * z)$$



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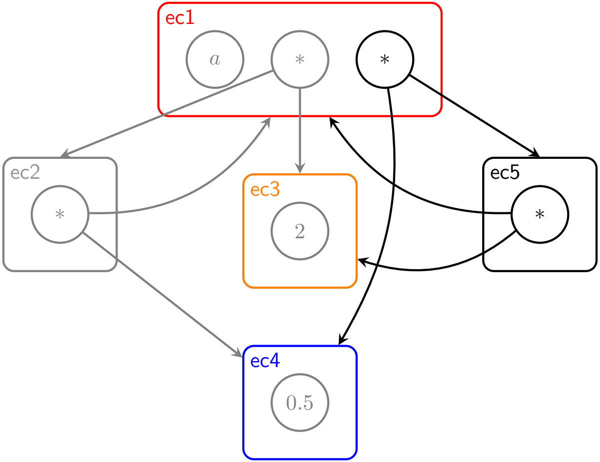

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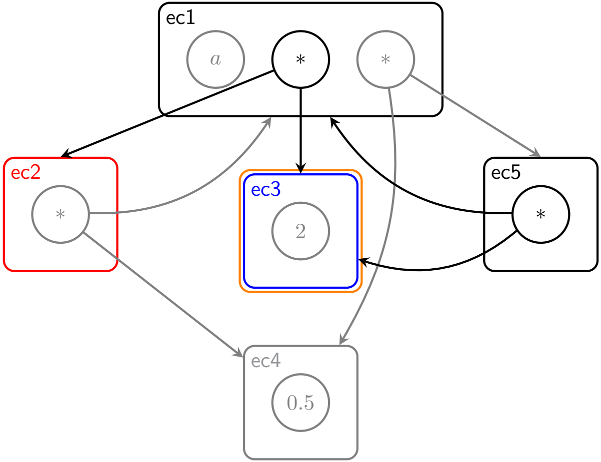

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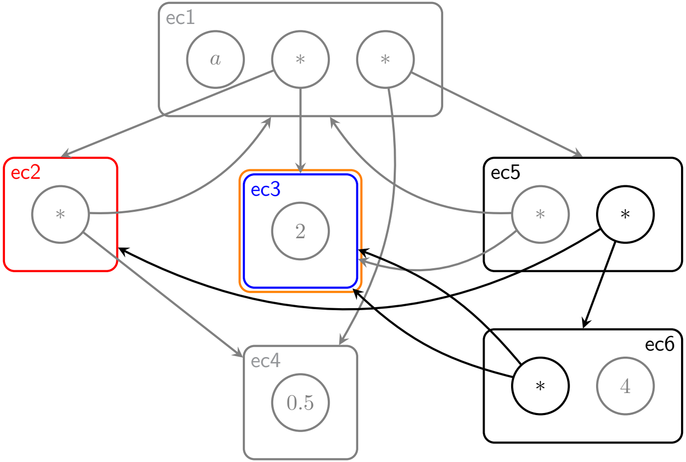

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Countermeasures

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Proposal

How about detecting redundant cycles in the egraph?

Challenge I: Saturation up to Redundancy

Superposition Saturation

Consider the first-order clause

$$\forall x.\ x = f(x, a)$$

We can generate infinitely many terms:

$$f(x, a), f(f(x, a), a), f(f(f(x, a), a), a), \ldots$$

Redundancy Criterion

A clause C is redundant if it is entailed by smaller clauses $\{C_1, \ldots, C_n\}$ according to a simplification ordering $\prec$:

$$\{C_1, \ldots, C_n\} \models C \quad \text{and} \quad \forall i : C_i \prec C$$

Challenge II: Egraphs Modulo User Propagator

Why it is useful?

- Some lemmas may depend on both sides of an equality. (e.g., $P(z|x) = 0.5 \longrightarrow P(z|x) = P(z)$)
- Simplify the egraph using domain knowledge.
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Challenges for Egraphs

- Explosion of number of terms.
- Managing explanations over egraph changes.

SAT with User Propagators

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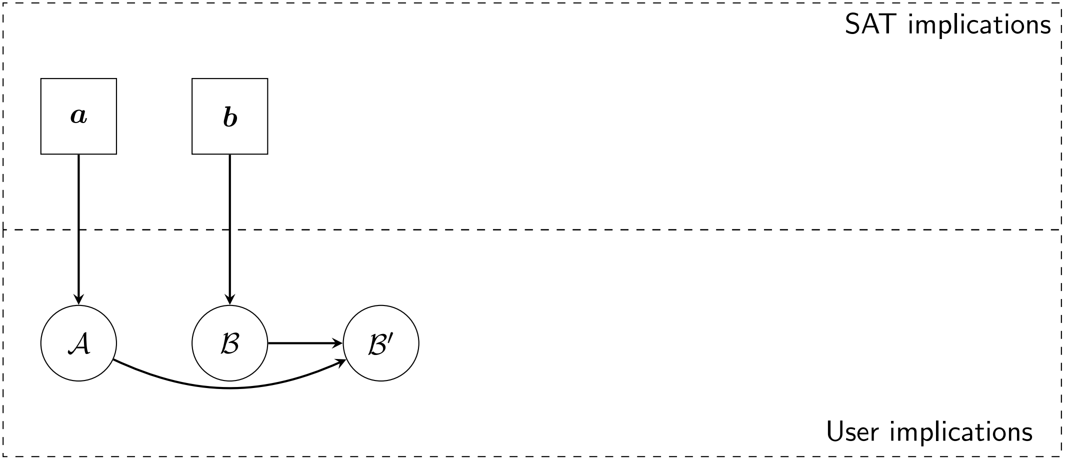

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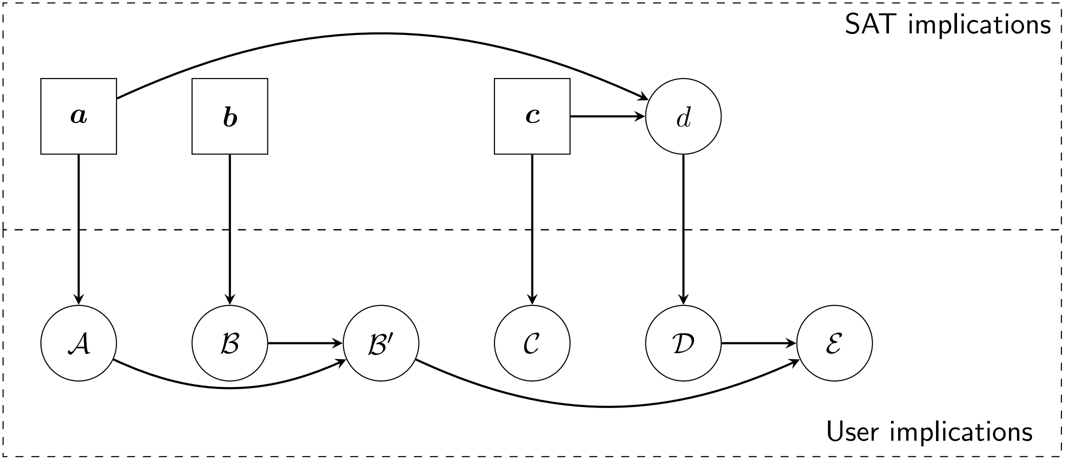

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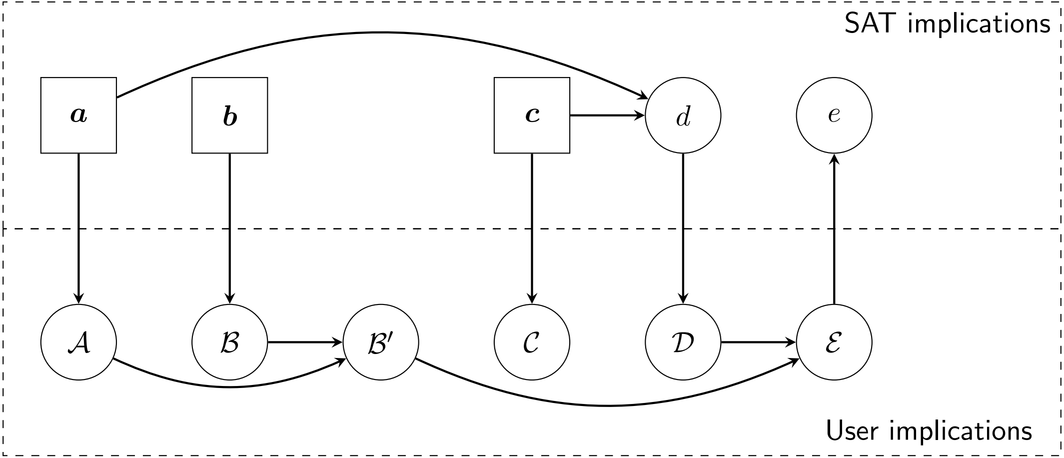
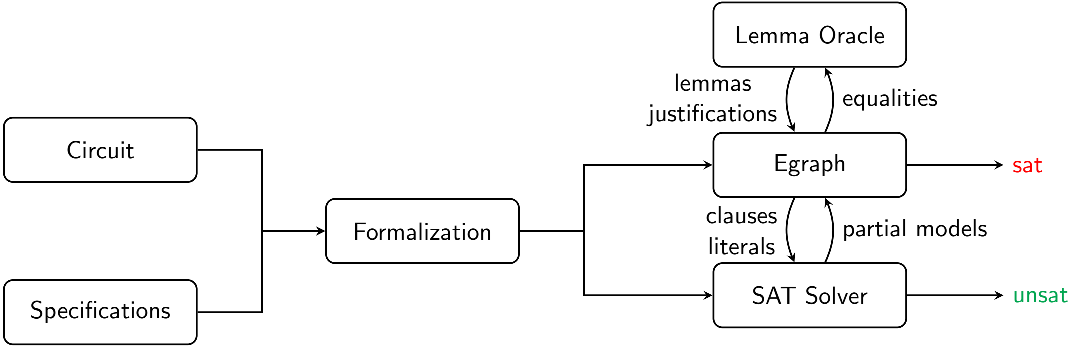


User Propagators with Egraphs



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Summary

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- Several challenges remain to be solved.
- We would like to use techniques from SMT/ATP solving within egraphs.

Challenges

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Discussion with the audience

- NRA: thoughts on saturation modulo redundancy?
- User propagators: any experience/ideas with egraphs?

Questions?

Thank you for your attention!

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