

Laziness in Automated Reasoning

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About me

Resume

- 2018 - 2021: Bachelor in Engineering at ULiège
- 2021 - 2023: Masters in Computer Science & Engineering at ULiège
- Winter 2022: Internship at TU Wien
- October 2023 - now: PhD at TU Wien

Research interests - Automated reasoning

- Propositional satisfiability (SAT)
- Satisfiability modulo theories (SMT)
- Saturation-based theorem proving (Vampire)

About me

Random Facts

- I speak Chinese and lived in China for a year
- I am Chess enthusiast
- I am a Game Master for Dungeons & Dragons
- I played the clarinet for many years
- ...

About me

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- I enjoy laziness

Relevance of Verification

Trust

Do you trust automatic systems?

Relevance of Verification

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Do you trust automatic systems? Engineers never make mistakes, right?

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(a) Metro Line 14 in Paris
- Fully automated

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(b) Pilotless planes?

Relevance of Verification

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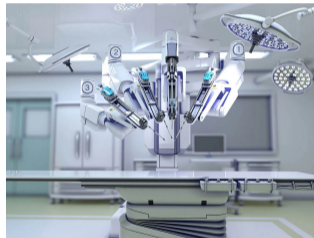
Do you trust automatic systems? Engineers never make mistakes, right?



(a) Metro Line 14 in Paris
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(b) Pilotless planes?



(c) Autonomous medical
robots?

Relevance of Verification

Trust

Do you trust automatic systems? Engineers never make mistakes, right?



(a) Metro Line 14 in Paris
- Fully automated and
verified



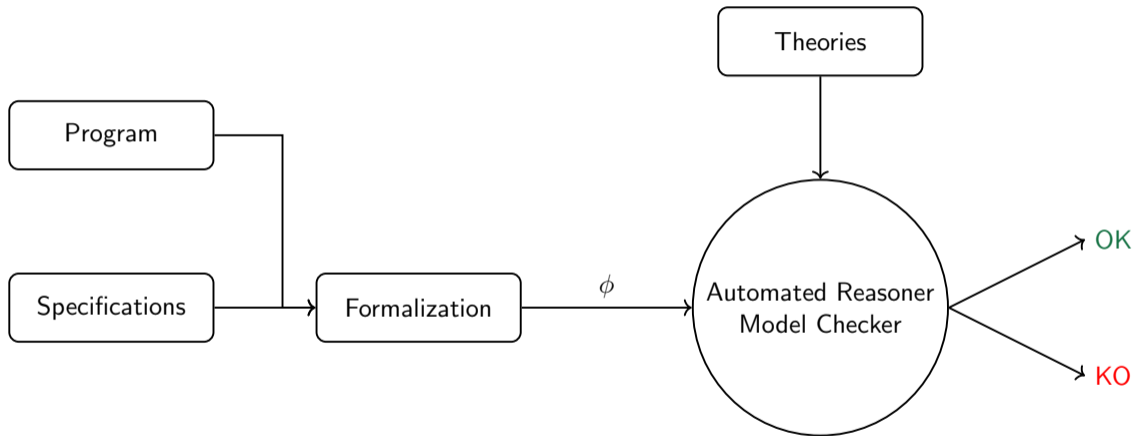
(b) Pilotless planes?



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What is it was verified?

Verification Pipeline



Opinions on Laziness

Definition (Laziness)

the quality of not being willing to work or use any effort. (Cambridge Dictionary)

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Laziness is nothing more than the habit of resting before you get tired.

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Laziness is the mother of all evil.

Sophocles

Laziness in Computer Science

Laziness is a programmer's best virtue.

Larry Wall

Laziness in Computer Science

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I choose a lazy person to do a hard job. Because a lazy person will find an easy way to do it.

Bill Gates

Laziness in Algorithms

Definition

An algorithm is lazy if it delays the computation of a value until that value is actually needed.

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Is laziness a good strategy?

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Well,... No



Is laziness a good strategy?

But sometimes,... Yes



Beautiful Properties of SAT Solving

Invariant (Backward compatible watched literals)

Consider the trail $\pi = \tau \cdot \omega$. For each clause $C \in F$ watched by the two distinct watched literals c_1, c_2 , we have $\neg c_1 \in \tau \Rightarrow [c_2 \in \pi \wedge \delta(c_2) \leq \delta(c_1)]$.

Beautiful Properties of SAT Solving



Chronological Backtracking - Sudoku Example

4	9			1		3		
				7			9	
5					9		4	
	3	4				7		
		2	6					3
			1		7	2		
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Chronological Backtracking - Sudoku Example

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Lazy Restoration of Invariants (([Coutelier et al. SAT 2024]))

Invariant (Lazy reimplication)

If the lazy reimplication reason $\lambda(\ell)$ of literal ℓ is defined, then the clause $\lambda(\ell)$ is a missed lower implication of ℓ . That is,

$$\begin{aligned}\lambda(\ell) \neq \blacksquare \Rightarrow & \quad \ell \in \pi \wedge \ell \in \lambda(\ell) \\ & \quad \wedge (\lambda(\ell) \setminus \{\ell\} \wedge \pi) \models \perp \\ & \quad \wedge \delta(\lambda(\ell) \setminus \{\ell\}) < \delta(\ell)\end{aligned}$$

Invariant (Lazy backtrack compatible watched literals)

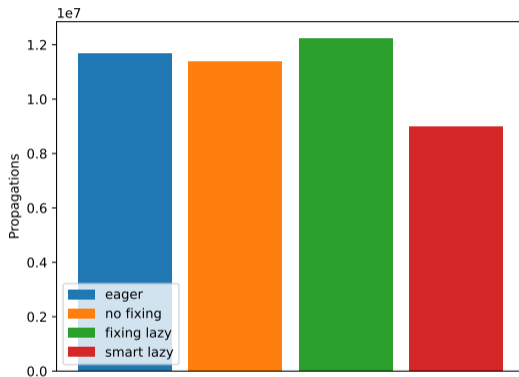
Consider the trail $\pi = \tau \cdot \omega$. For each clause $C \in F$ watched by the two distinct watched literals c_1, c_2 , we have

$$\neg c_1 \in \tau \Rightarrow \left(c_2 \in \pi \wedge (\delta(c_2) \leq \delta(c_1) \vee \delta(\lambda(c_2) \setminus \{c_2\}) \leq \delta(c_1)) \right)$$

Lazy Restoration of Invariants



Results ([Coutelier et al. SAT 2024])



Number of propagations performed by NapSAT on 100 random 3-SAT problems with 250 variables.

Lazy Meter



Laziness in Team Work

Lazy Meter

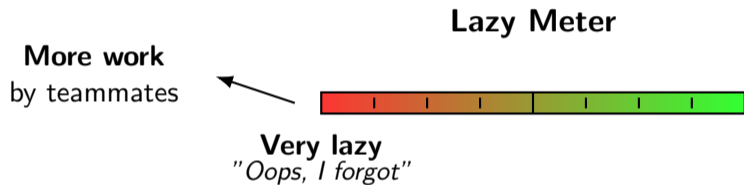


Lazy Meter

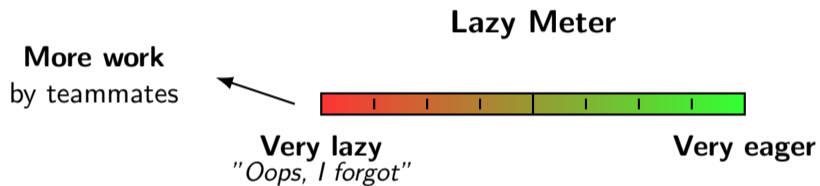


Very lazy
"Oops, I forgot"

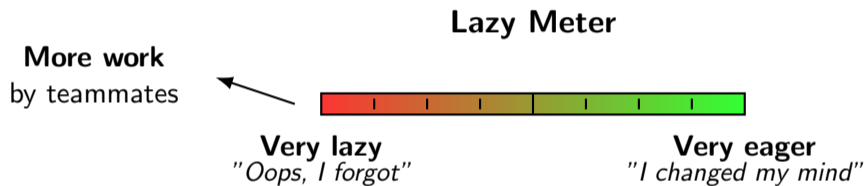
Laziness in Team Work



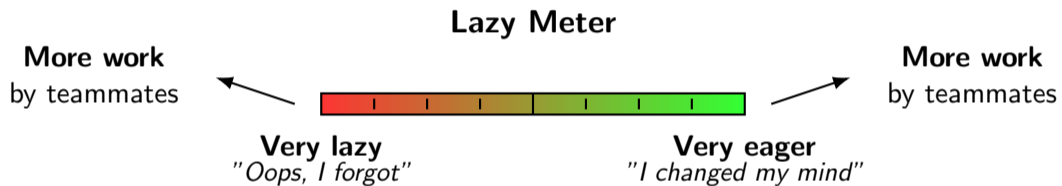
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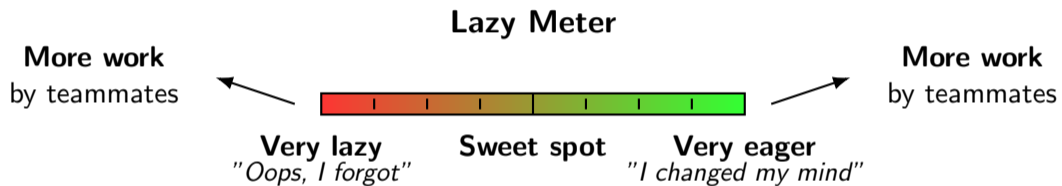
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Laziness in Team Work



SAT-SMT Team Work

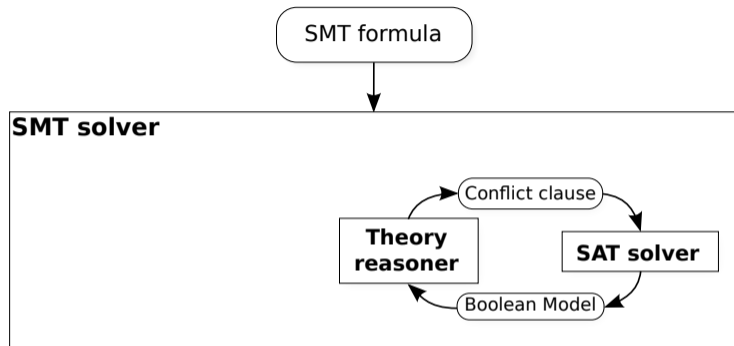


Figure: Schematic of a typical SMT solver.

Project

- **Goal:** Improve the efficiency of SAT and SMT solvers by using Chronological Backtracking.
- **Partners:** TU Wien, ULiège
- **Duration:** 3 years
- **Funding:** Fondation Gochet

Components and Functionalities of MODULARiT

- NAPSAT - SAT solver developed based on [Coutelier, Thesis 2024] (TU Wien)
- Congruence closure algorithm (ULiège)
- Term manager (ULiège)
- On going: Quantifier instantiation (ULiège)
- On going: Probability Reasoning (TU Wien)

Conclusion

- Automated Reasoning is an important field in Computer Science
- Laziness can be a good strategy if used correctly
- A lazy approach to invariant maintenance can improve SAT solvers

Future Work

- Analyze the impact of Chronological Backtracking on SMT solvers
- Search collaboration strategies between SAT and SMT solvers

Publication List

- Robin Coutelier. *Chronological vs. Non-Chronological Backtracking in SMT*. Master Thesis 2023.
- Robin Coutelier, Jakob Rath, Michael Rawson, and Laura Kovács. *SAT-Based Subsumption Resolution*. In CADE 2023.
- Robin Coutelier, Mathias Fleury, and Laura Kovács. *Lazy Reimplication in Chronological Backtracking*. In SAT 2024.
- Robin Coutelier. *To Link or Not to Link?* In PoS 2024.
- Robin Coutelier, Jakob Rath, Michael Rawson, Armin Biere, and Laura Kovács. *SAT Solving for Variants of First-Order Subsumption*. In FMSD 2024.