Less Aggressive Backtracking of Expensive SAT Literals

BIRS Workshop

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Acknowledgements

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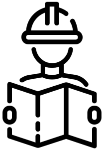
Introduction



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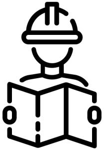

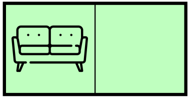

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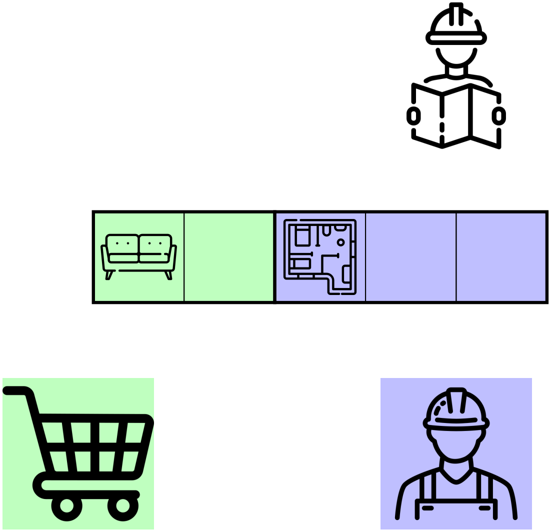

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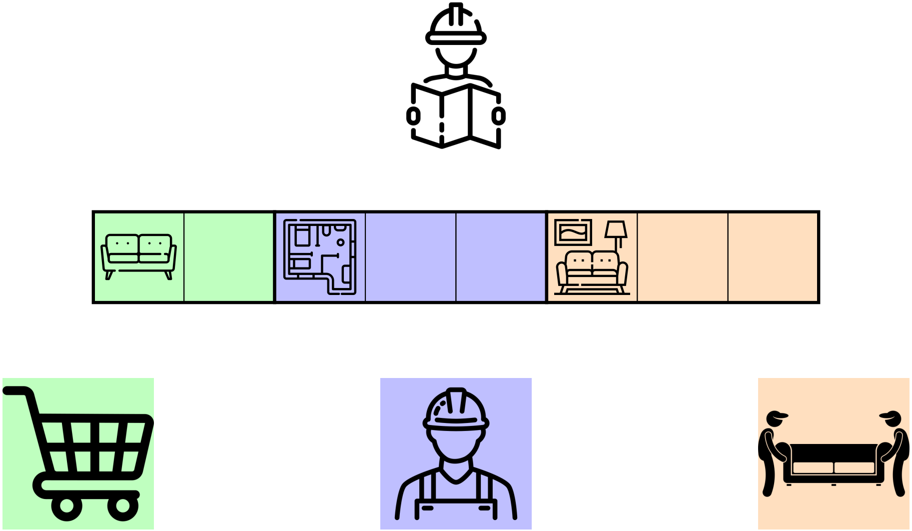

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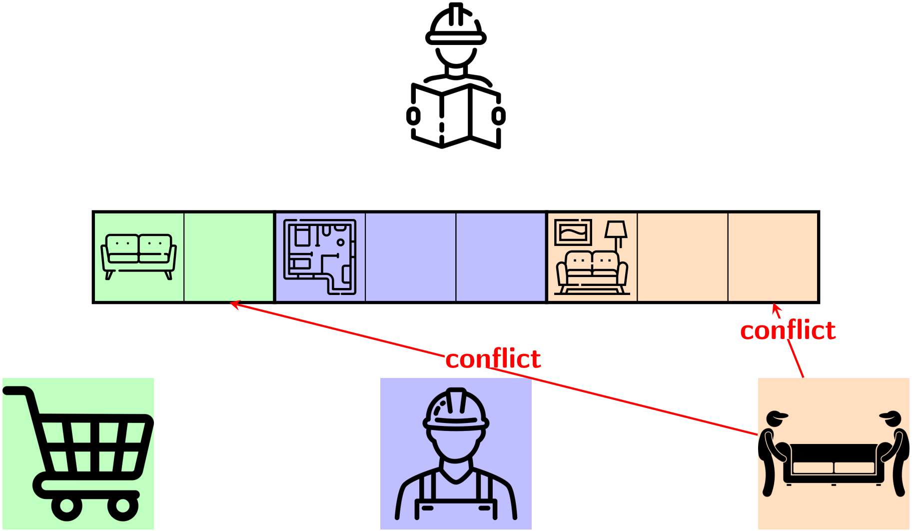

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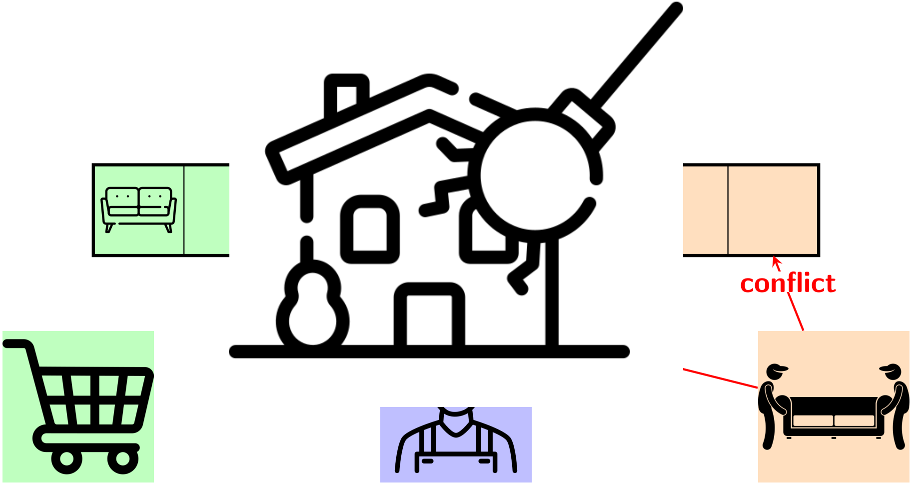

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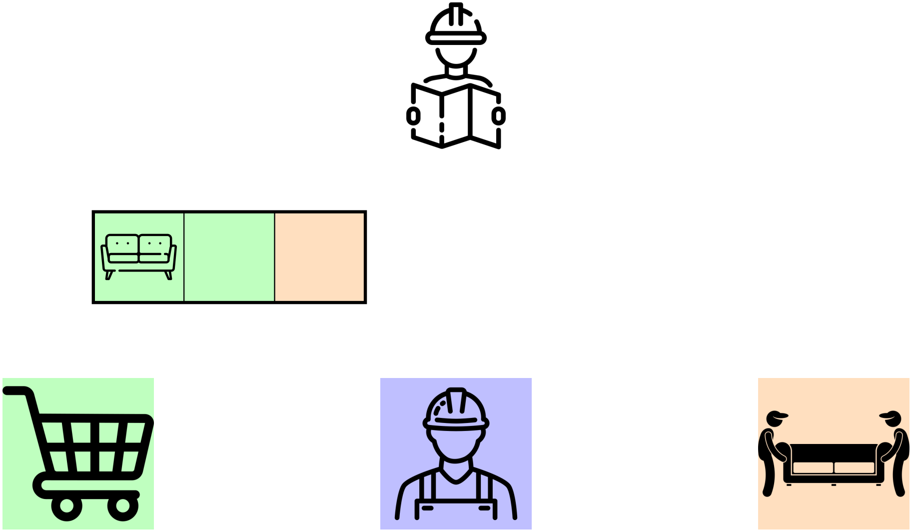

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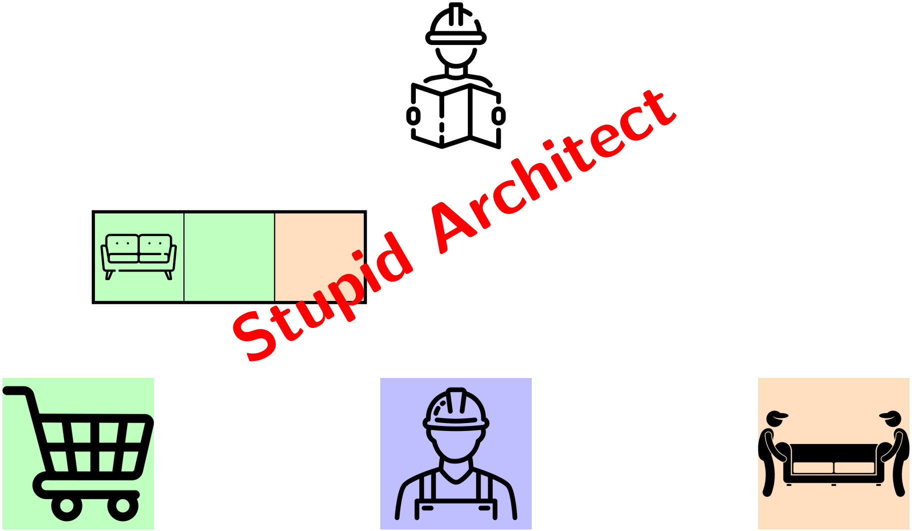
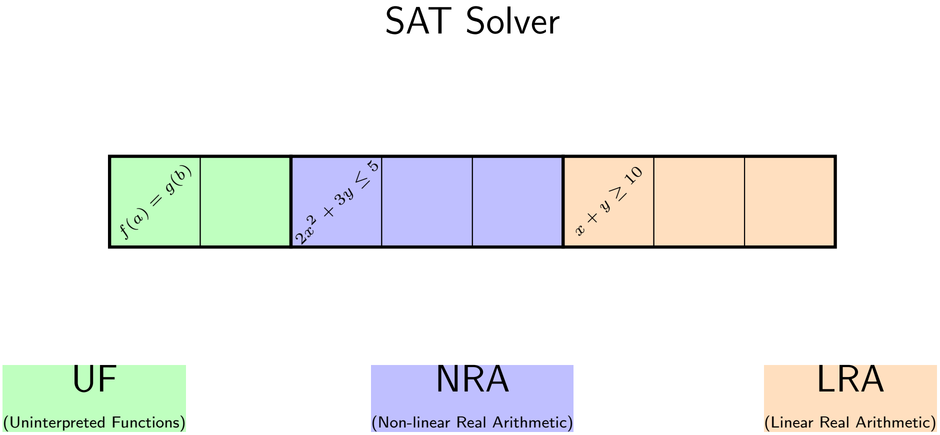
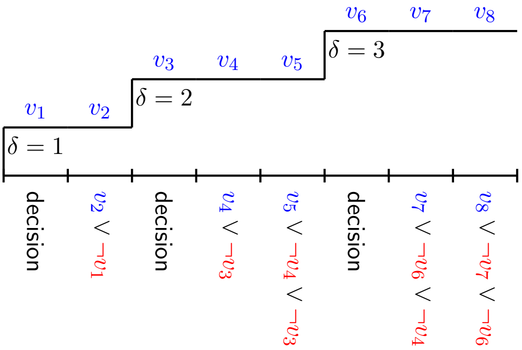


SMT Architecture



Non-Chronological Backtracking – Trail



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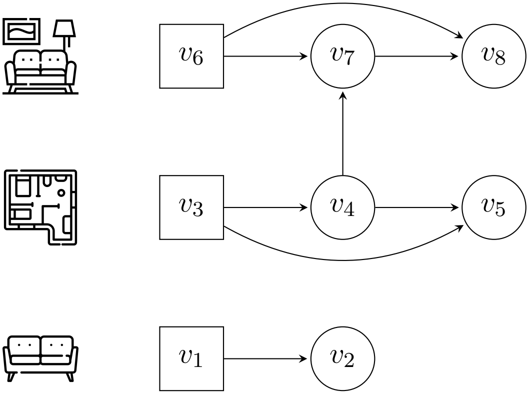

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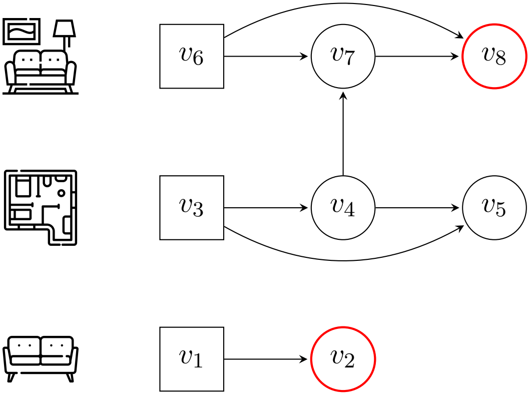

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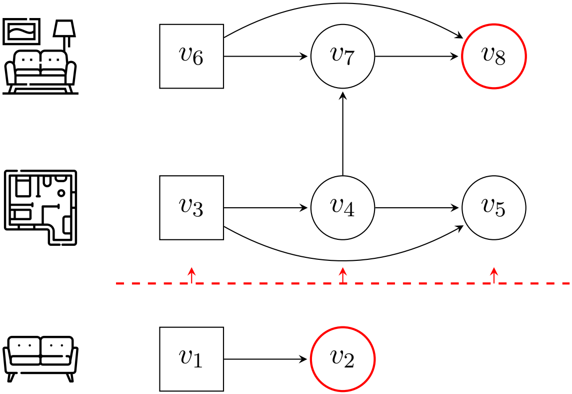

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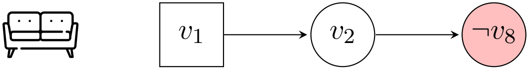
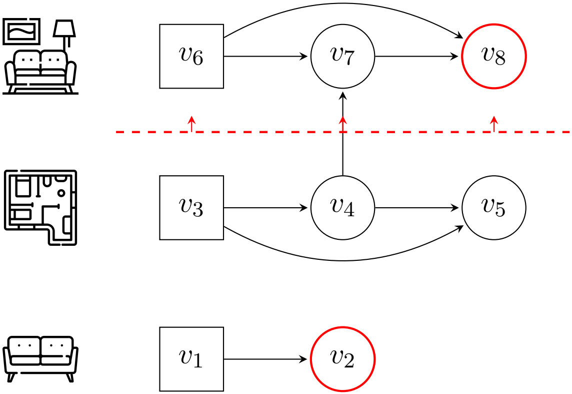


Chronological Backtracking [NR18, MB19, Nad22, CFK24]



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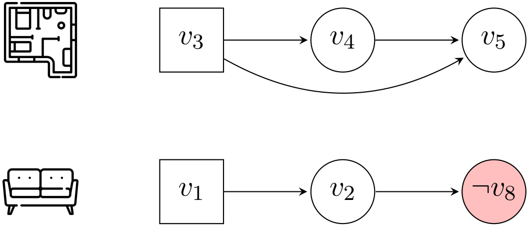

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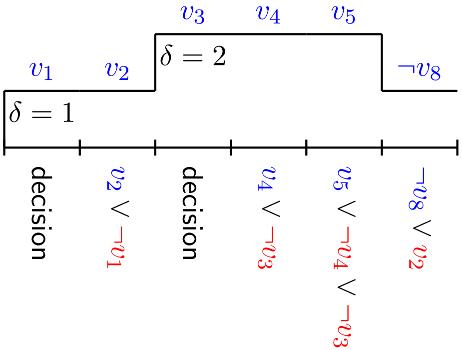

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To be fast

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To be general

- The SAT solver does not know about theories
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To be smart

- Aggressive backjumping can undo a lot of useless work
- Maybe learn fewer clauses

Invariants in NCB and CB

Consider the trail $\pi = \tau \cdot \omega$. For each clause $C \in F$ watched by c_1, c_2 in $\mathrm{WL}(c_1)$, we have

Invariant (NCB Watched Literals)

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Invariant (CB Watched Literals)

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Smarter Architects

Extended Interface

We allow the user to provide a **cost** $\zeta(\ell)$ for each literal ℓ.

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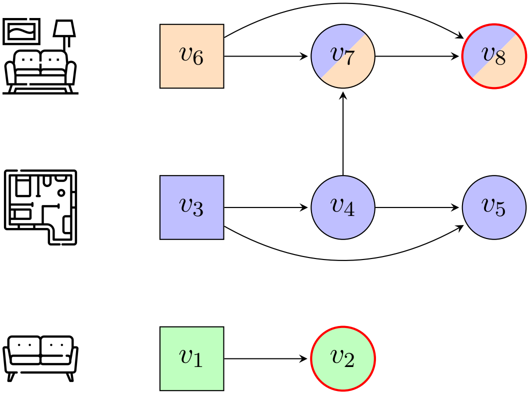
New Backtracking Scheme

Graph Backtracking (GB) searches the cheapest set of literals to backtrack to resolve a conflict. We use the implication graph to find such a set.

Graph Backtracking

Backtrack options

option 1:
option 2:
option 3:



Graph Backtracking

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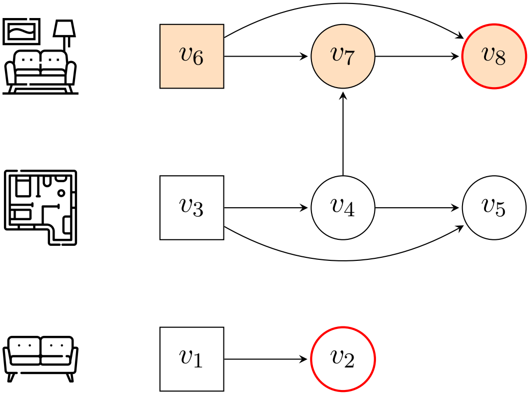

Graph Backtracking

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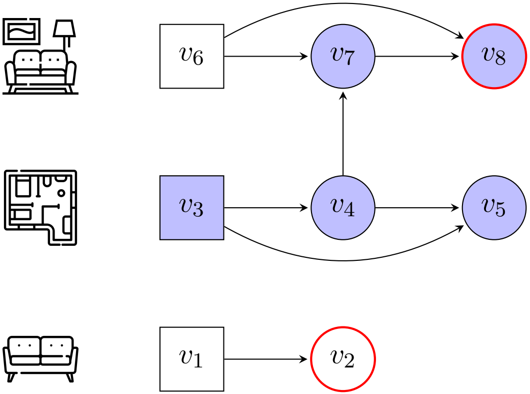

Graph Backtracking

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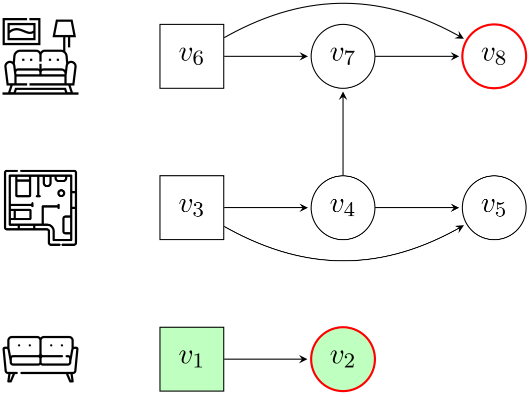

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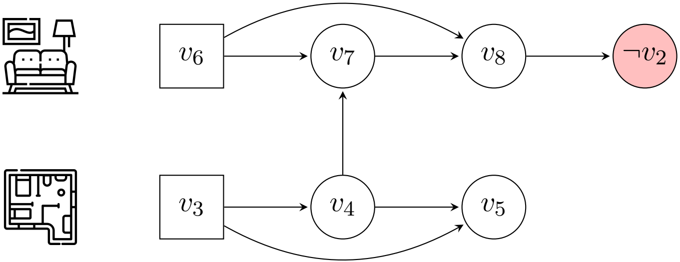


Invariants in NCB, CB and GB

Consider the trail $\pi = \tau \cdot \omega$. For each clause $C \in F$ watched by c_1, c_2 and blocked by b in $\mathrm{WL}(c_1)$, we have

Invariant (NCB Watched Literals)

$$\neg c_1 \in \tau \Rightarrow c_2 \in \pi$$

Invariant (CB Watched Literals)

$$\neg c_1 \in \tau \Rightarrow \big(c_2 \in \pi \wedge \delta(c_2) \leq \delta(c_1)\big)$$

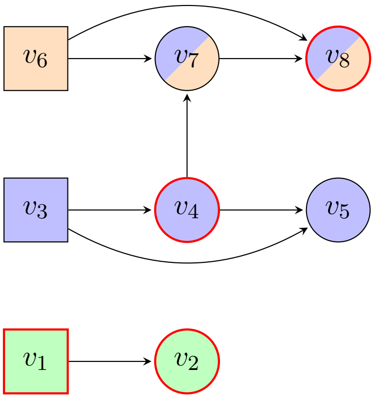
Invariant (GB Watched Literals)

$$\neg c_1 \in \tau \Rightarrow \big(c_2 \in \pi \wedge \gamma(c_2) \subseteq \eta(c_1)\big)$$

1UIP with GB

Conflicting clause: $\neg v_1 \vee \neg v_2 \vee \neg v_4 \vee \neg v_8$

Backtrack
options



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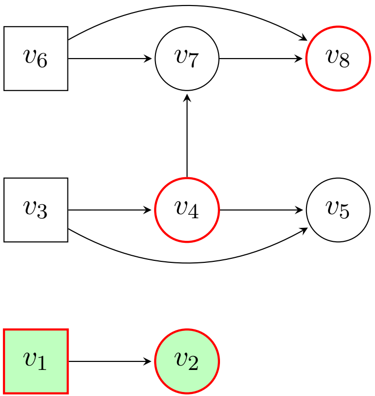

1UIP with GB

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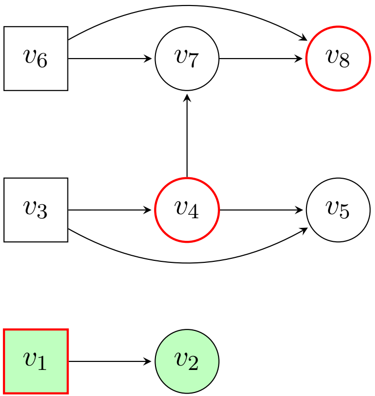
Backtrack options

option 1: ◯ $\boldsymbol{\neg v_1} \vee \neg v_4 \vee \neg v_8$



1UIP with GB

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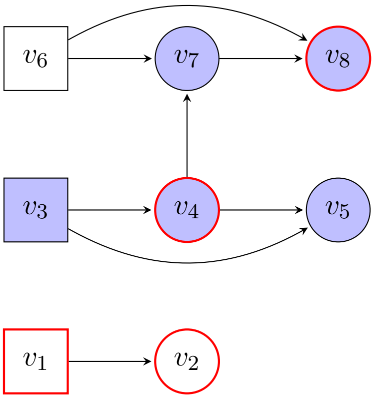
Backtrack options

option 1: (green circle) $\boldsymbol{\neg v_1} \vee \neg v_4 \vee \neg v_8$

option 2: (blue circle)



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Conflicting clause: $\neg v_1 \lor \neg v_2 \lor \neg v_4 \lor \neg v_8$

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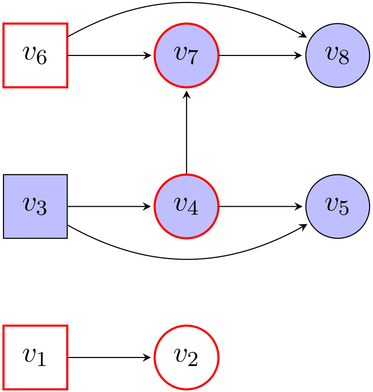
option 2: ◯



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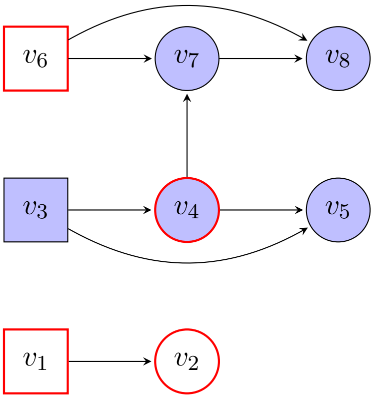
option 1: $\boldsymbol{\neg v_1} \lor \neg v_4 \lor \neg v_8$

option 2: $\neg v_1 \lor \neg v_2 \lor \boldsymbol{\neg v_4} \lor \neg v_6$



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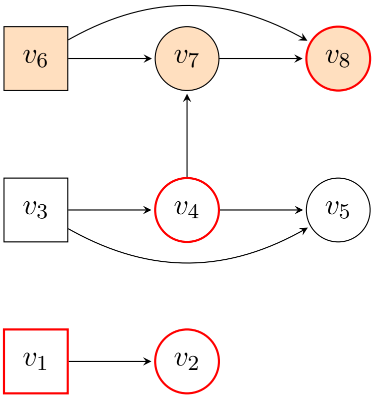

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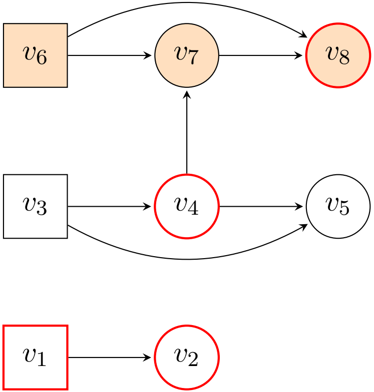
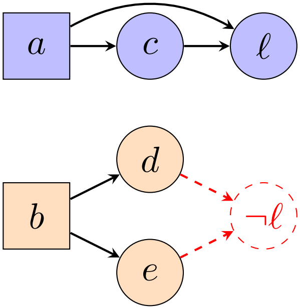


Challenge I: Termination



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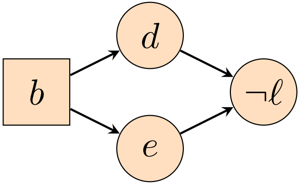

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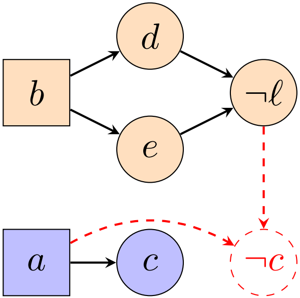

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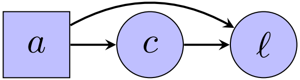

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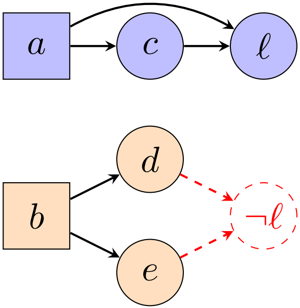

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Challenge I: Safeguards

Why this happens

- Arbitrary cost function $\zeta(\ell)$
- No guarantee to learn a new clause after backtracking (like in CB [MB19])

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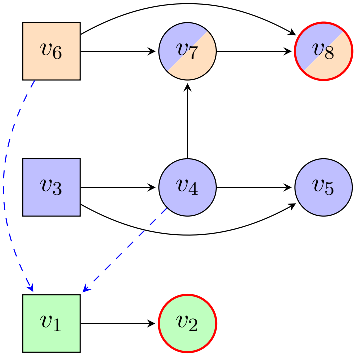
Solution

- If possible to learn a clause: choose the chunk that allows to learn the clause with minimum total cost
- Select latest decision otherwise (like in CB)

Challenge II: Redundant Implications

Backtrack options

option 1: (green)
option 2: (blue)
option 3: (orange)



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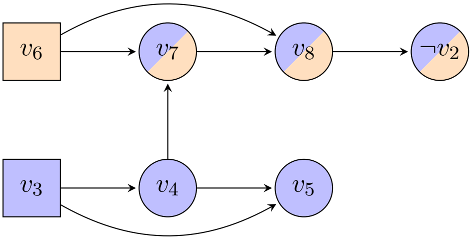

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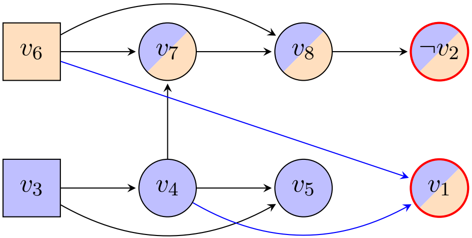

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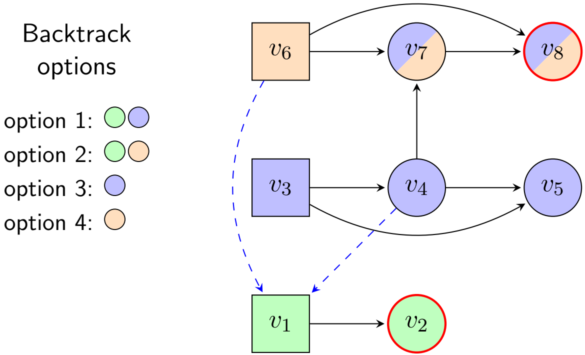
C_6 &= \neg v_8 \vee \neg v_2 \\
C_7 &= v_1 \vee \neg v_4 \vee \neg v_6
\end{aligned}
$$

Challenge II: Redundant Implications



$$C_1 = v_2 \lor \neg v_1$$
$$C_2 = v_4 \lor \neg v_3$$
$$C_3 = v_5 \lor \neg v_4 \lor \neg v_3$$
$$C_4 = v_7 \lor \neg v_6 \lor \neg v_4$$
$$C_5 = v_8 \lor \neg v_7 \lor \neg v_6$$
$$C_6 = \neg v_8 \lor \neg v_2$$
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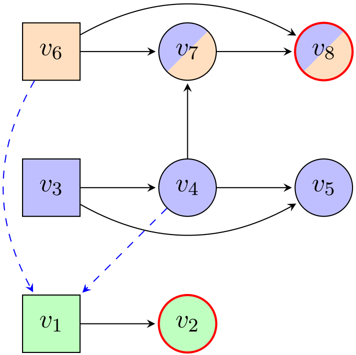
Challenge II: Redundant Implications

Backtrack options

~~option 1:~~ (green, blue)
~~option 2:~~ (green, orange)
option 3: (blue)
option 4: (orange)



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$$C_2 = v_4 \vee \neg v_3$$
$$C_3 = v_5 \vee \neg v_4 \vee \neg v_3$$
$$C_4 = v_7 \vee \neg v_6 \vee \neg v_4$$
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Opportunities

Immediate vs. Exhaustive Conflict Repair [BF25]

When a conflict is detected, we can either

- immediately backtrack to resolve it (like NCB and CB)
- or gather all possible conflicts and resolve them in one big backtrack step

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The cost function $\zeta(\ell)$ can be dynamically adjusted based on cached information from the user.

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Multiple Clause Learning

We can attempt learning multiple clauses from the same conflict, and pick the best one according to the cost function.
(it does not work well in practice though)

Empirical Results

Option	Time (s)	Sync $\times 10^3$	Propagation $\times 10^6$
NCB	1.11 ± 3.15	567.98 ± 925.2	2.56 ± 4.29
CB	1.13 ± 5.41	508.73 ± 930.64	2.47 ± 4.58
GB	1.93 ± 5.43	$\mathbf{358.65} \pm \mathbf{571.43}$	1.84 ± 3.04
NCB+ECR	14.88 ± 68.77	511.46 ± 788.33	8.09 ± 13.05
CB+ECR	16.51 ± 73.67	487.29 ± 799.73	8.65 ± 14.65
GB+ECR	32.55 ± 131.72	$\mathbf{319.18} \pm \mathbf{510.48}$	7.38 ± 12.5

Table: Experiments on 1000 graph coloring instances `kcolor 3 gmn 400 920`. Average time, number of synchronizations and propagations. Standard deviation is shown after the $\pm$ symbol.

Conclusion

Summary

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Future Work

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- Relax SMT stack discipline into multi stack
- Implement GB in CVC5 [BBB+22]

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Thank you for your attention!

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