Lazy Reimplication in Chronological Backtracking

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Acknowledgements

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and Technology Fund

Contributions of this Work

- Invariant framework for chronological backtracking
- Classification of chronological backtracking algorithms
- Introduction of lazy strong chronological backtracking
- Implementation of lazy strong chronological backtracking in NapSAT, CaDiCaL and Glucose
- Experimental evaluation of our method

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Dichotomy of Chronological Bactracking

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CDCL
greedy backtracking?

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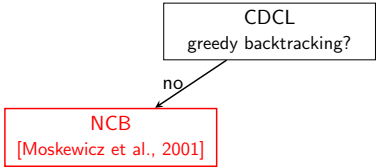

Conflict Driven Clause Learning [Moskewicz et al., 2001]

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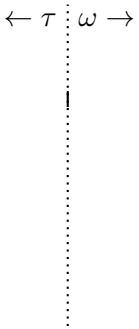

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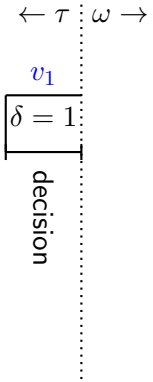

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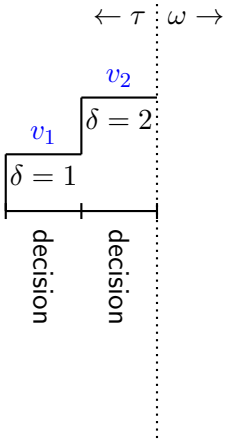

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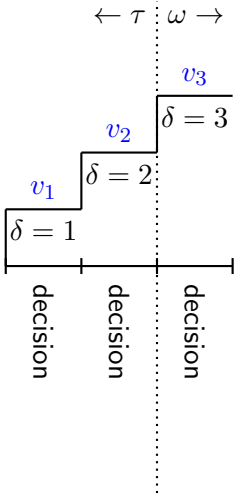

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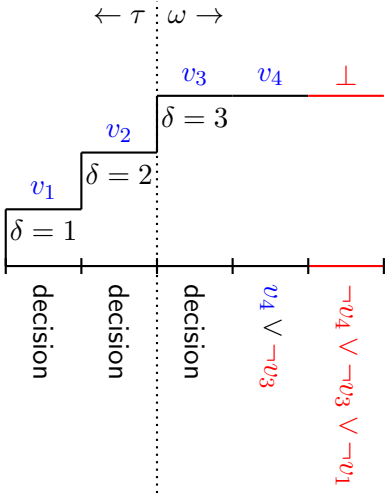

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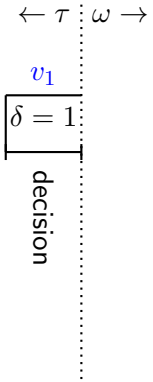

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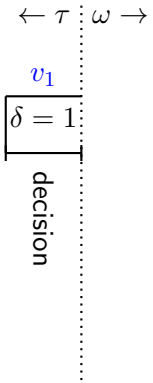

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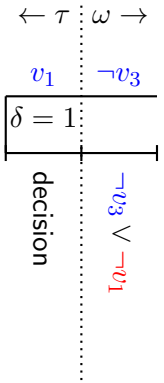

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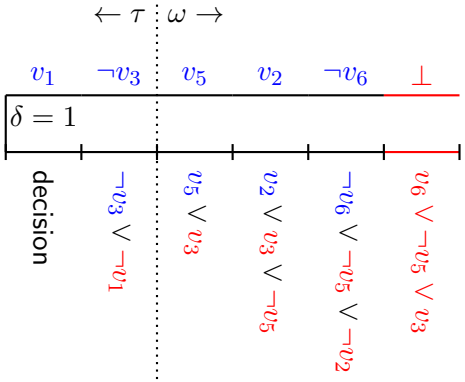

Invariants in Non-Chronological Backtracking CDCL

Invariant (Implied literals)

If a literal ℓ is in the trail π, then ℓ is either a decision literal or ℓ is implied by π and its reason $\rho(\ell)$. That is,

$$\forall \ell \in \pi.\ \ell \in \pi^d \ \vee\ [\ell \in \rho(\ell)\ \wedge\ [\rho(\ell) \setminus \{\ell\} \wedge \pi] \vDash \bot]\,.$$

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The trail π is a topological order of the implication graph.

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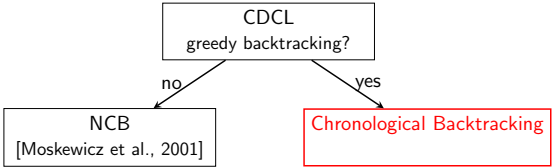
Invariant (Strong watched literals)

Consider the trail $\pi = \tau \cdot \omega$. For each clause $C \in F$ watched by the two distinct watched literals c_1, c_2, we have $\neg c_1 \in \tau \Rightarrow c_2 \in \pi$.

Dichotomy of Chronological Bactracking



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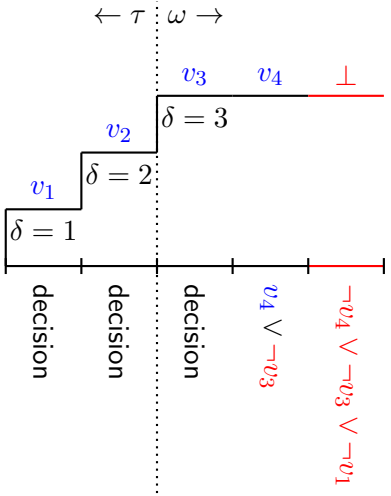

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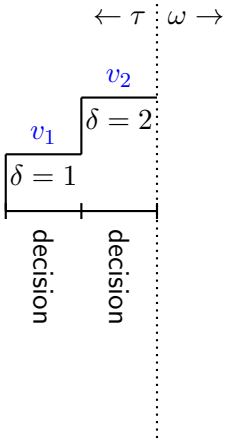

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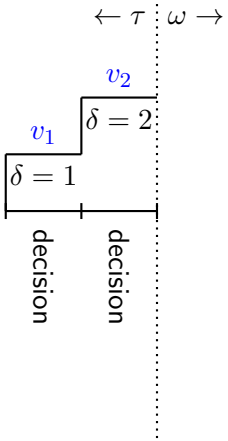

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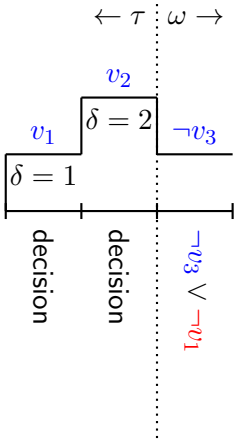

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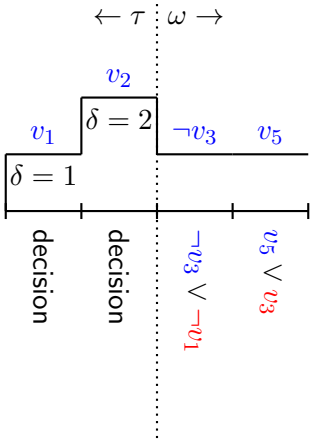

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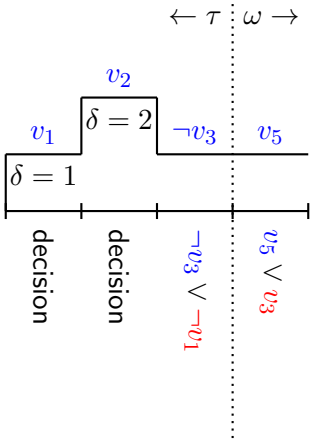

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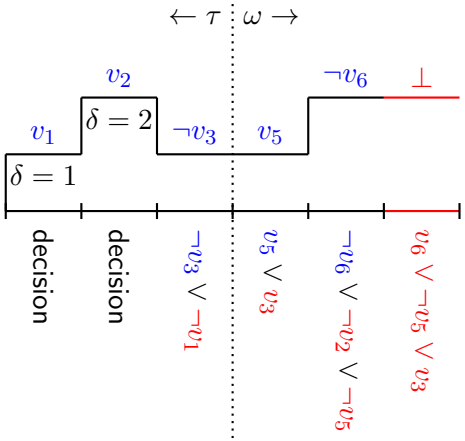

Weak Chronological Backtracking – [Coutelier, 2023]

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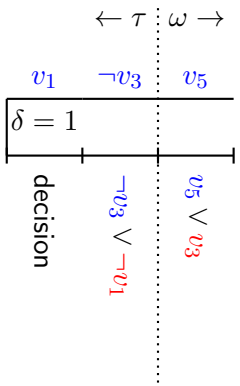

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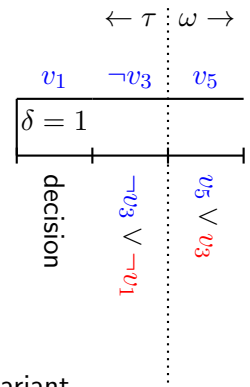
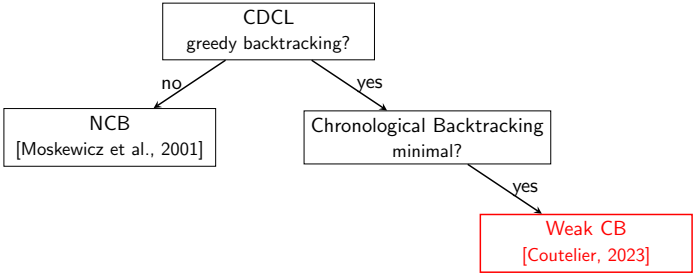


C_4 violates the strong watched literals invariant.

$\neg v_3 \in \tau \Rightarrow v_2 \in \pi$

Dichotomy of Chronological Bactracking



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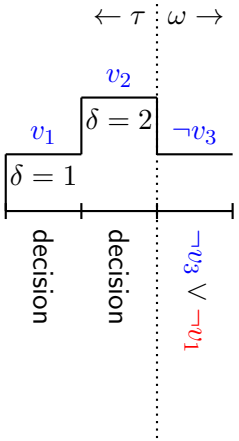

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\end{aligned}
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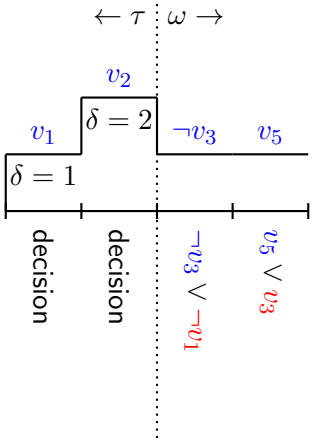


Weak Invariants in WCB

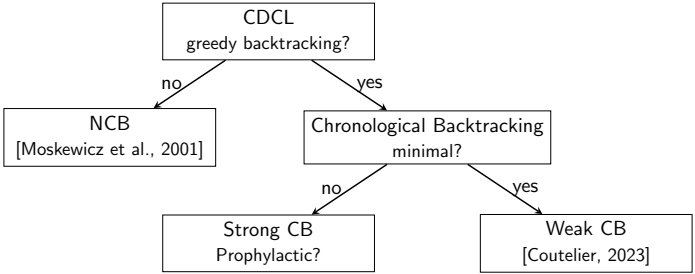
Invariant (Strong watched literals)

Consider the trail $\pi = \tau \cdot \omega$. For each clause $C \in F$ watched by the two distinct watched literals c_1, c_2, we have $\neg c_1 \in \tau \Rightarrow c_2 \in \pi$.

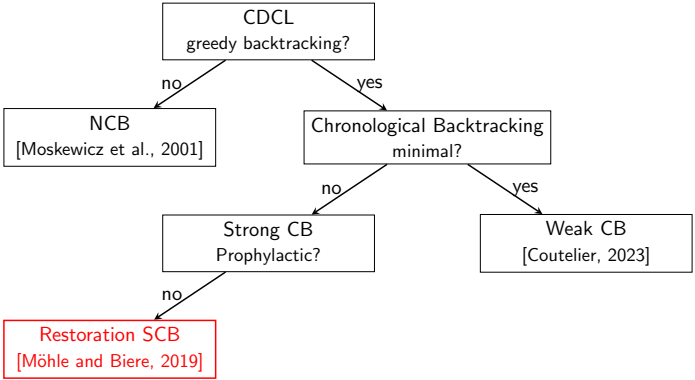
Invariant (Weak watched literals)

Consider the trail $\pi = \tau \cdot \omega$. For each clause $C \in F$ watched by the two distinct watched literals c_1, c_2, we have $\neg c_1 \in \tau \Rightarrow \neg c_2 \notin \tau$.

Dichotomy of Chronological Bactracking

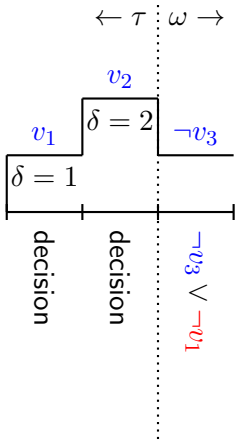


Dichotomy of Chronological Bactracking



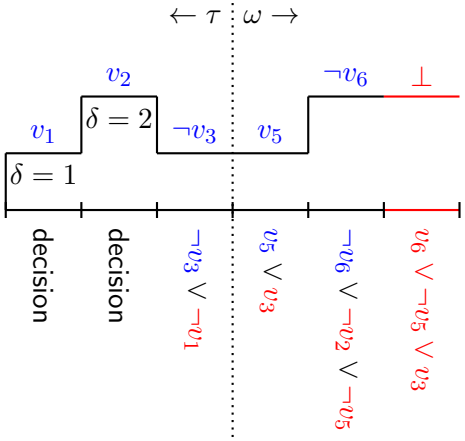
Restoration Strong Chronological Backtracking – [Möhle and Biere, 2019]

$$
\begin{aligned}
C_1 &= \neg v_3 \vee v_4 \\
C_2 &= \neg v_3 \vee \neg v_4 \vee \neg v_1 \\
C_3 &= v_5 \vee v_3 \\
C_4 &= v_2 \vee v_3 \vee \neg v_5 \\
C_5 &= \neg v_5 \vee \neg v_6 \vee \neg v_2 \\
C_6 &= \neg v_5 \vee v_3 \vee v_6 \\
C_7 &= \neg v_3 \vee \neg v_1
\end{aligned}
$$



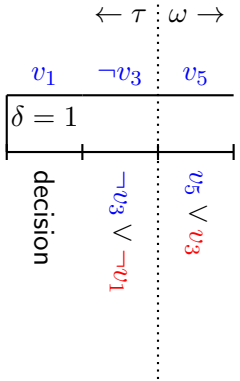
Restoration Strong Chronological Backtracking – [Möhle and Biere, 2019]

$$
\begin{aligned}
C_1 &= \neg\underline{v_3} \vee \underline{v_4} \\
C_2 &= \neg\underline{v_3} \vee \neg\underline{v_4} \vee \neg v_1 \\
C_3 &= \underline{v_5} \vee \underline{v_3} \\
C_4 &= \underline{v_2} \vee \underline{v_3} \vee \neg v_5 \\
C_5 &= \neg\underline{v_6} \vee \neg\underline{v_2} \vee \neg v_5 \\
C_6 &= \underline{v_6} \vee \neg\underline{v_5} \vee v_3 \\
C_7 &= \neg\underline{v_3} \vee \neg\underline{v_1}
\end{aligned}
$$



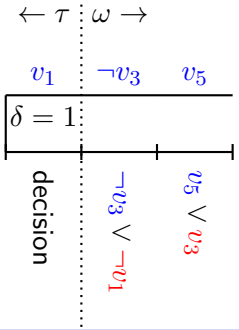
Restoration Strong Chronological Backtracking – [Möhle and Biere, 2019]

$$
\begin{aligned}
C_1 &= \neg v_3 \vee \quad v_4 \\
C_2 &= \neg v_3 \vee \neg v_4 \vee \neg v_1 \\
C_3 &= \quad v_5 \vee \quad v_3 \\
C_4 &= \quad v_2 \vee \quad v_3 \vee \neg v_5 \\
C_5 &= \neg v_2 \vee \neg v_6 \vee \neg v_5 \\
C_6 &= \quad v_6 \vee \neg v_5 \vee \quad v_3 \\
C_7 &= \neg v_3 \vee \neg v_1
\end{aligned}
$$



Restoration Strong Chronological Backtracking – [Möhle and Biere, 2019]

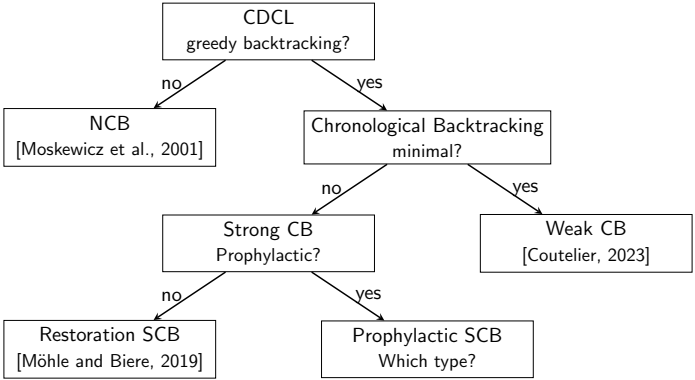
$$
\begin{aligned}
C_1 &= \neg v_3 \vee \ v_4 \\
C_2 &= \neg v_3 \vee \neg v_4 \vee \neg v_1 \\
C_3 &= \ v_5 \vee \ v_3 \\
C_4 &= \ v_2 \vee \ v_3 \vee \neg v_5 \\
C_5 &= \neg v_2 \vee \neg v_6 \vee \neg v_5 \\
C_6 &= \ v_6 \vee \neg v_5 \vee \ v_3 \\
C_7 &= \neg v_3 \vee \neg v_1
\end{aligned}
$$



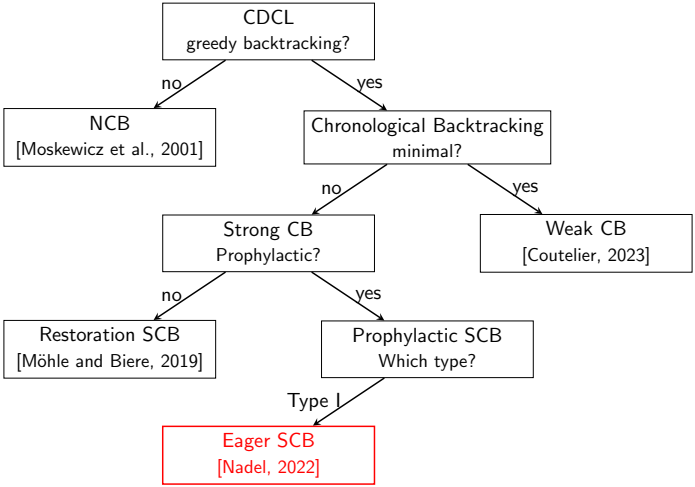
Invariant (Strong watched literals)

Consider the trail $\pi = \tau \cdot \omega$. For each clause $C \in F$ watched by the two distinct watched literals c_1, c_2, we have $\neg c_1 \in \tau \Rightarrow c_2 \in \pi$.

Dichotomy of Chronological Bactracking

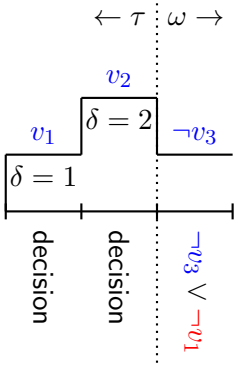


Dichotomy of Chronological Bactracking



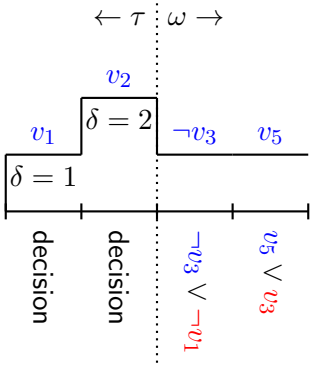
Eager Strong Chronological Backtracking – [Nadel, 2022]

$$
\begin{aligned}
C_1 &= \neg v_3 \vee \quad v_4 \\
C_2 &= \neg v_3 \vee \neg v_4 \vee \neg v_1 \\
C_3 &= \quad v_5 \vee \quad v_3 \\
C_4 &= \quad v_2 \vee \quad v_3 \vee \neg v_5 \\
C_5 &= \neg v_5 \vee \neg v_6 \vee \neg v_2 \\
C_6 &= \neg v_5 \vee \quad v_3 \vee \quad v_6 \\
C_7 &= \neg v_3 \vee \neg v_1
\end{aligned}
$$



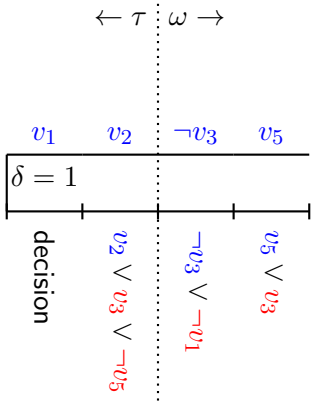
Eager Strong Chronological Backtracking – [Nadel, 2022]

$$
\begin{aligned}
C_1 &= \neg v_3 \vee \quad v_4 \\
C_2 &= \neg v_3 \vee \neg v_4 \vee \neg v_1 \\
C_3 &= \quad v_5 \vee \quad v_3 \\
C_4 &= \quad v_2 \vee \quad v_3 \vee \neg v_5 \\
C_5 &= \neg v_5 \vee \neg v_6 \vee \neg v_2 \\
C_6 &= \neg v_5 \vee \quad v_3 \vee \quad v_6 \\
C_7 &= \neg v_3 \vee \neg v_1
\end{aligned}
$$



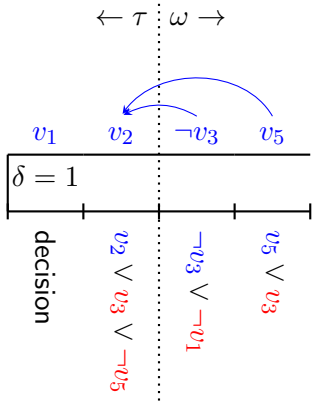
Eager Strong Chronological Backtracking – [Nadel, 2022]

$$
\begin{aligned}
C_1 &= \neg v_3 \vee \quad v_4 \\
C_2 &= \neg v_3 \vee \neg v_4 \vee \neg v_1 \\
C_3 &= \quad v_5 \vee \quad v_3 \\
C_4 &= \quad v_2 \vee \quad v_3 \vee \neg v_5 \\
C_5 &= \neg v_5 \vee \neg v_6 \vee \neg v_2 \\
C_6 &= \neg v_5 \vee \quad v_3 \vee \quad v_6 \\
C_7 &= \neg v_3 \vee \neg v_1
\end{aligned}
$$



Eager Strong Chronological Backtracking – [Nadel, 2022]

$$
\begin{aligned}
C_1 &= \neg v_3 \vee \quad v_4 \\
C_2 &= \neg v_3 \vee \neg v_4 \vee \neg v_1 \\
C_3 &= \quad v_5 \vee \quad v_3 \\
C_4 &= \quad v_2 \vee \quad v_3 \vee \neg v_5 \\
C_5 &= \neg v_5 \vee \neg v_6 \vee \neg v_2 \\
C_6 &= \neg v_5 \vee \quad v_3 \vee \quad v_6 \\
C_7 &= \neg v_3 \vee \neg v_1
\end{aligned}
$$

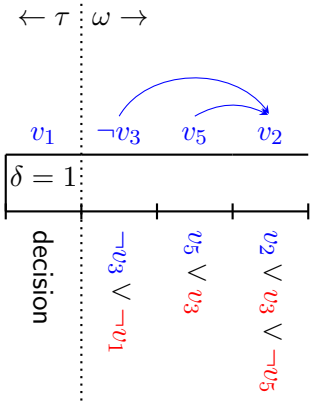


Invariant (Topological order)

The trail π is a topological order of the implication graph.

Eager Strong Chronological Backtracking – [Nadel, 2022]

$$
\begin{aligned}
C_1 &= \neg v_3 \vee \quad v_4 \\
C_2 &= \neg v_3 \vee \neg v_4 \vee \neg v_1 \\
C_3 &= \quad v_5 \vee \quad v_3 \\
C_4 &= \quad v_2 \vee \quad v_3 \vee \neg v_5 \\
C_5 &= \neg v_5 \vee \neg v_6 \vee \neg v_2 \\
C_6 &= \neg v_5 \vee \quad v_3 \vee \quad v_6 \\
C_7 &= \neg v_3 \vee \neg v_1
\end{aligned}
$$



Invariant (Topological order)

The trail π is a topological order of the implication graph.

Backtrack Compatible Watched Literals

Invariant (Strong watched literals)

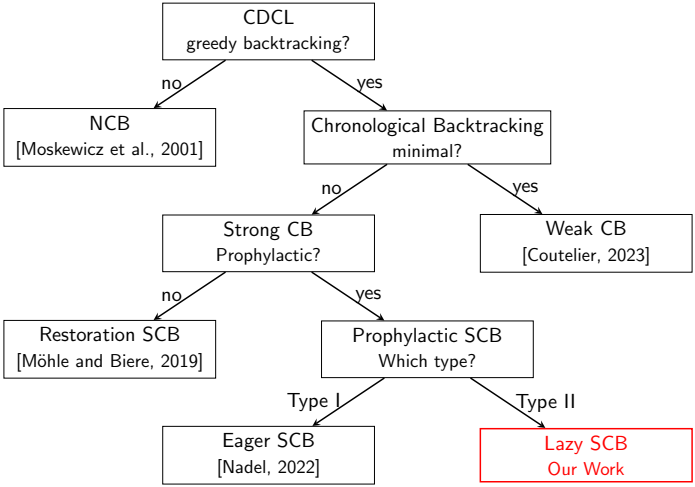
Consider the trail $\pi = \tau \cdot \omega$. *For each clause* $C \in F$ *watched by the two distinct watched literals* c_1, c_2, *we have* $\neg c_1 \in \tau \Rightarrow c_2 \in \pi$.

Invariant (Backward compatible watched literals)

Consider the trail $\pi = \tau \cdot \omega$. *For each clause* $C \in F$ *watched by the two distinct watched literals* c_1, c_2, *we have* $\neg c_1 \in \tau \Rightarrow [c_2 \in \pi \wedge \delta(c_2) \leq \delta(c_1)]$.

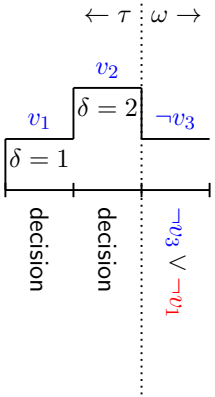
If backward compatible watched literals holds, no clause will be unit (by τ) after a backtracking.

Dichotomy of Chronological Bactracking



Lazy Strong Chronological Backtracking – Our work

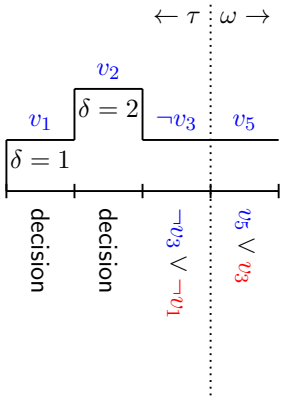
$$
\begin{aligned}
C_1 &= \neg v_3 \vee \quad v_4 \\
C_2 &= \neg v_3 \vee \neg v_4 \vee \neg v_1 \\
C_3 &= \quad v_5 \vee \quad v_3 \\
C_4 &= \quad v_2 \vee \quad v_3 \vee \neg v_5 \\
C_5 &= \neg v_5 \vee \neg v_6 \vee \neg v_2 \\
C_6 &= \neg v_5 \vee \quad v_3 \vee \quad v_6 \\
C_7 &= \neg v_3 \vee \neg v_1
\end{aligned}
$$



Var	λ
v_1	
v_2	
v_3	
v_4	
v_5	
v_6	

Lazy Strong Chronological Backtracking – Our work

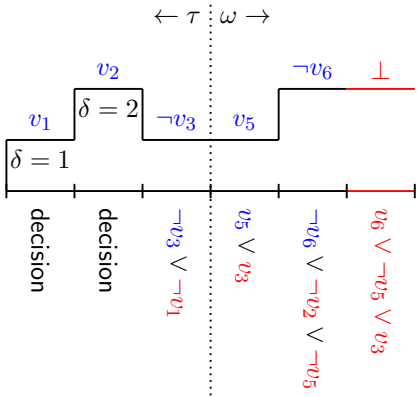
$$
\begin{aligned}
C_1 &= \neg\underline{v_3} \vee \underline{v_4} \\
C_2 &= \neg\underline{v_3} \vee \neg\underline{v_4} \vee \neg v_1 \\
C_3 &= \underline{v_5} \vee \underline{v_3} \\
C_4 &= \underline{v_2} \vee \underline{v_3} \vee \neg v_5 \\
C_5 &= \neg\underline{v_6} \vee \neg\underline{v_5} \vee \neg v_2 \\
C_6 &= \underline{v_6} \vee \neg\underline{v_5} \vee v_3 \\
C_7 &= \neg\underline{v_3} \vee \neg\underline{v_1}
\end{aligned}
$$



Var	λ
v_1	
v_2	C_4
v_3	
v_4	
v_5	
v_6	

Lazy Strong Chronological Backtracking – Our work

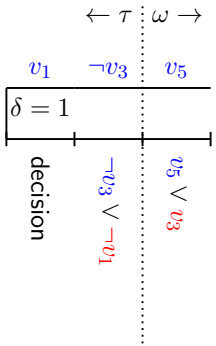
$$
\begin{aligned}
C_1 &= \neg v_3 \vee \ v_4 \\
C_2 &= \neg v_3 \vee \neg v_4 \vee \neg v_1 \\
C_3 &= \ v_5 \vee \ v_3 \\
C_4 &= \ v_2 \vee \ v_3 \vee \neg v_5 \\
C_5 &= \neg v_6 \vee \neg v_2 \vee \neg v_5 \\
C_6 &= \ v_6 \vee \neg v_5 \vee \ v_3 \\
C_7 &= \neg v_3 \vee \neg v_1
\end{aligned}
$$



Var	λ
v_1	
v_2	C_4
v_3	
v_4	
v_5	
v_6	

Lazy Strong Chronological Backtracking – Our work

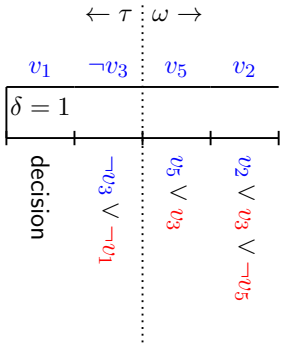
$$
\begin{aligned}
C_1 &= \neg v_3 \vee \quad v_4 \\
C_2 &= \neg v_3 \vee \neg v_4 \vee \neg v_1 \\
C_3 &= \quad v_5 \vee \quad v_3 \\
C_4 &= \quad v_2 \vee \quad v_3 \vee \neg v_5 \\
C_5 &= \neg v_2 \vee \neg v_6 \vee \neg v_5 \\
C_6 &= \quad v_6 \vee \neg v_5 \vee \quad v_3 \\
C_7 &= \neg v_3 \vee \neg v_1
\end{aligned}
$$



Var	λ
v_1	
v_2	C_4
v_3	
v_4	
v_5	
v_6	

Lazy Strong Chronological Backtracking – Our work

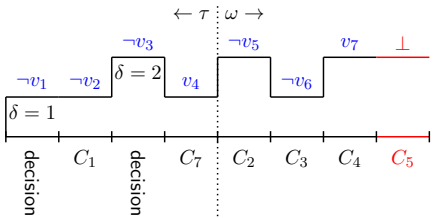
$$
\begin{aligned}
C_1 &= \neg v_3 \vee \ v_4 \\
C_2 &= \neg v_3 \vee \neg v_4 \vee \neg v_1 \\
C_3 &= \ v_5 \vee \ v_3 \\
C_4 &= \ v_2 \vee \ v_3 \vee \neg v_5 \\
C_5 &= \neg v_6 \vee \neg v_2 \vee \neg v_5 \\
C_6 &= \ v_6 \vee \neg v_5 \vee \ v_3 \\
C_7 &= \neg v_3 \vee \neg v_1
\end{aligned}
$$



Var	λ
v_1	
v_2	
v_3	
v_4	
v_5	
v_6	

Improved Conflict Analysis – Our Work

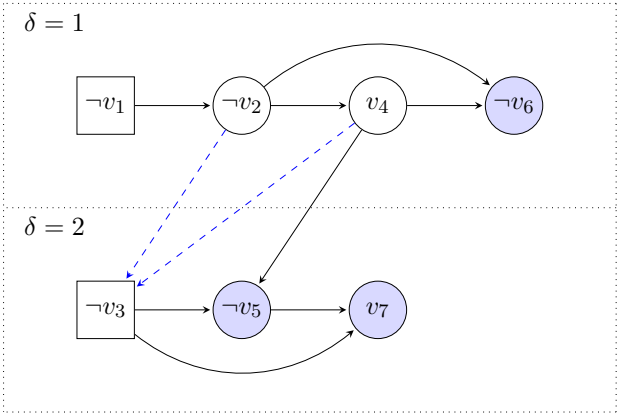
$$
\begin{aligned}
C_1 &= \neg\underline{v_2} \vee \underline{v_1} \\
C_2 &= \neg\underline{v_5} \vee \underline{v_3} \vee \neg v_4 \\
C_3 &= \neg\underline{v_6} \vee \underline{v_2} \vee \neg v_4 \\
C_4 &= \underline{v_7} \vee \underline{v_5} \vee v_3 \\
C_5 &= \underline{v_5} \vee \neg\underline{v_7} \vee v_6 \\
C_6 &= \neg\underline{v_3} \vee \neg\underline{v_4} \vee v_2 \\
C_7 &= \underline{v_4} \vee \underline{v_2}
\end{aligned}
$$



Var	λ
v_1	
v_2	
v_3	C_6
v_4	
v_5	
v_6	
v_7	

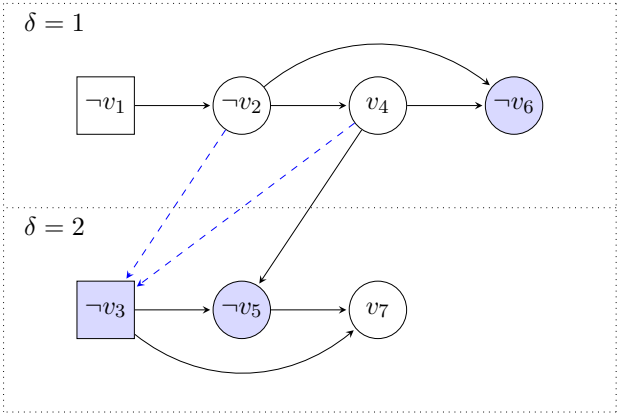
Better Conflict Analysis – Our Work

$$
\begin{aligned}
C_1 &= \neg v_2 \vee \quad v_1 \\
C_2 &= \neg v_5 \vee \quad v_3 \vee \neg v_4 \\
C_3 &= \neg v_6 \vee \quad v_2 \vee \neg v_4 \\
C_4 &= \quad v_7 \vee \quad v_5 \vee \quad v_3 \\
C_5 &= \quad v_5 \vee \neg v_7 \vee \quad v_6 \\
C_6 &= \neg v_3 \vee \neg v_4 \vee \quad v_2 \\
C_7 &= \quad v_4 \vee \quad v_2
\end{aligned}
$$



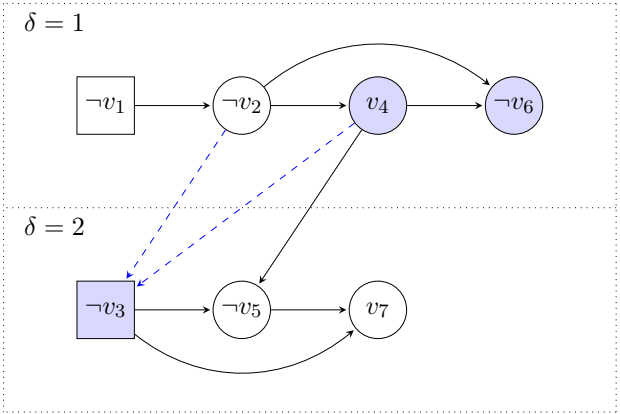
Better Conflict Analysis – Our Work

$$
\begin{aligned}
C_1 &= \neg v_2 \vee \quad v_1 \\
C_2 &= \neg v_5 \vee \quad v_3 \vee \neg v_4 \\
C_3 &= \neg v_6 \vee \quad v_2 \vee \neg v_4 \\
C_4 &= \quad v_7 \vee \quad v_5 \vee \quad v_3 \\
C_5 &= \quad v_5 \vee \neg v_7 \vee \quad v_6 \\
C_6 &= \neg v_3 \vee \neg v_4 \vee \quad v_2 \\
C_7 &= \quad v_4 \vee \quad v_2
\end{aligned}
$$



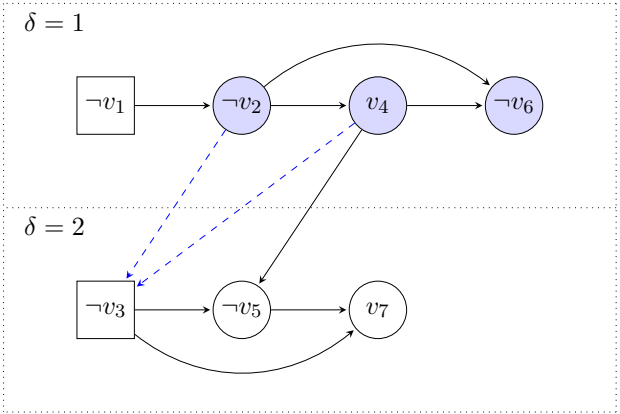
Better Conflict Analysis – Our Work

$$
\begin{aligned}
C_1 &= \neg v_2 \vee \quad v_1 \\
C_2 &= \neg v_5 \vee \quad v_3 \vee \neg v_4 \\
C_3 &= \neg v_6 \vee \quad v_2 \vee \neg v_4 \\
C_4 &= \quad v_7 \vee \quad v_5 \vee \quad v_3 \\
C_5 &= \quad v_5 \vee \neg v_7 \vee \quad v_6 \\
C_6 &= \neg v_3 \vee \neg v_4 \vee \quad v_2 \\
C_7 &= \quad v_4 \vee \quad v_2
\end{aligned}
$$



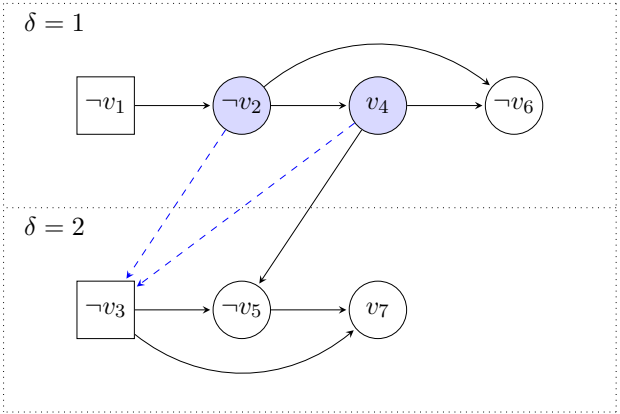
Better Conflict Analysis – Our Work

$$
\begin{aligned}
C_1 &= \neg v_2 \vee \quad v_1 \\
C_2 &= \neg v_5 \vee \quad v_3 \vee \neg v_4 \\
C_3 &= \neg v_6 \vee \quad v_2 \vee \neg v_4 \\
C_4 &= \quad v_7 \vee \quad v_5 \vee \quad v_3 \\
C_5 &= \quad v_5 \vee \neg v_7 \vee \quad v_6 \\
C_6 &= \neg v_3 \vee \neg v_4 \vee \quad v_2 \\
C_7 &= \quad v_4 \vee \quad v_2
\end{aligned}
$$



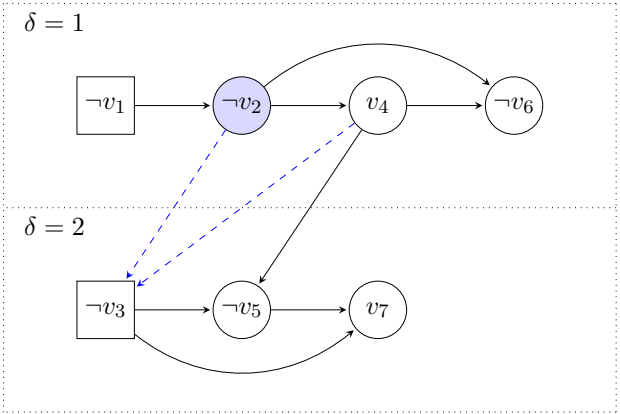
Better Conflict Analysis – Our Work

$$
\begin{aligned}
C_1 &= \neg v_2 \vee \quad v_1 \\
C_2 &= \neg v_5 \vee \quad v_3 \vee \neg v_4 \\
C_3 &= \neg v_6 \vee \quad v_2 \vee \neg v_4 \\
C_4 &= \quad v_7 \vee \quad v_5 \vee \quad v_3 \\
C_5 &= \quad v_5 \vee \neg v_7 \vee \quad v_6 \\
C_6 &= \neg v_3 \vee \neg v_4 \vee \quad v_2 \\
C_7 &= \quad v_4 \vee \quad v_2
\end{aligned}
$$



Better Conflict Analysis – Our Work

$$
\begin{aligned}
C_1 &= \neg v_2 \vee \quad v_1 \\
C_2 &= \neg v_5 \vee \quad v_3 \vee \neg v_4 \\
C_3 &= \neg v_6 \vee \quad v_2 \vee \neg v_4 \\
C_4 &= \quad v_7 \vee \quad v_5 \vee \quad v_3 \\
C_5 &= \quad v_5 \vee \neg v_7 \vee \quad v_6 \\
C_6 &= \neg v_3 \vee \neg v_4 \vee \quad v_2 \\
C_7 &= \quad v_4 \vee \quad v_2
\end{aligned}
$$



Invariants in LSCB

Invariant (Lazy reimplication)

If the lazy reimplication reason $\lambda(\ell)$ of literal ℓ is defined, then the clause $\lambda(\ell)$ is a missed lower implication of ℓ. That is,

$$\begin{aligned}\lambda(\ell) \neq \blacksquare \;\Rightarrow\quad & \ell \in \pi \wedge\ \ell \in \lambda(\ell) \\ & \wedge \left(\lambda(\ell) \setminus \{\ell\} \wedge \pi\right) \vDash \bot \\ & \wedge\ \delta(\lambda(\ell) \setminus \{\ell\}) < \delta(\ell)\end{aligned}$$

Invariant (Lazy backtrack compatible watched literals)

Consider the trail $\pi = \tau \cdot \omega$. For each clause $C \in F$ watched by the two distinct watched literals c_1, c_2, we have

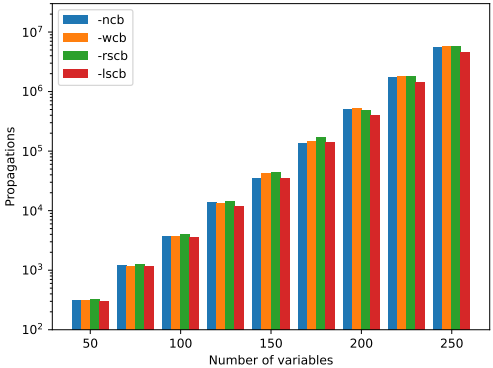
$$\neg c_1 \in \tau \Rightarrow \Big(c_2 \in \pi \wedge \big(\delta(c_2) \leq \delta(c_1) \vee \delta(\lambda(c_2) \setminus \{c_2\}) \leq \delta(c_1)\big)\Big)$$

Experimental Setup

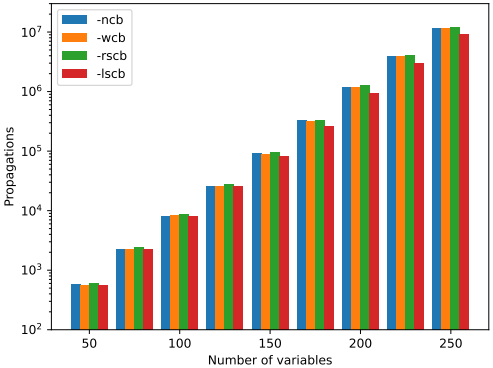
Table: Implementation of methods in this work.

	NCB	WCB	RSCB	ESCB	LSCB
NapSAT	✓	✓	✓	✗	✓
CaDiCaL	✓	✗	✓	✓	✓
Glucose	✓	✗	✗	✗	✓

Results in NapSAT



Satisfiable instances



Unsatisfiable instances

Average total number of propagations for Uniform Random 3-SAT problems

Results in CaDiCaL

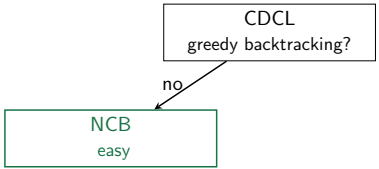
Table: Number of solved instances by different variants of strong backtracking on the SC2023 competition, using a $5000\,\mathrm{s}$ timeout

CaDiCaL version	solved	PAR-2 ($\times 10^3$)
base-CaDiCaL = RSCB	**248**	**4.09**
LSCB, Analyze-2 and minimization	246	4.16
ESCB	245	4.16
LSCB and Analyze-2	246	4.19
NCB	247	4.19
LSCB and Analyze-1	242	4.24

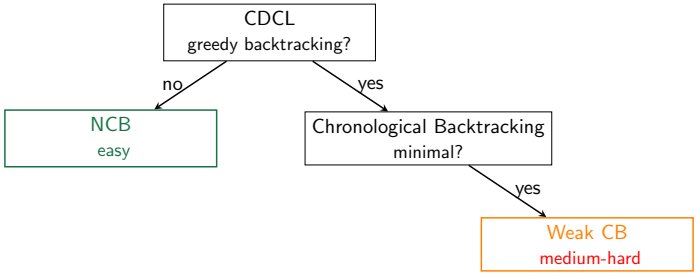
Backtracking Algorithm Implementation Complexity

CDCL
greedy backtracking?

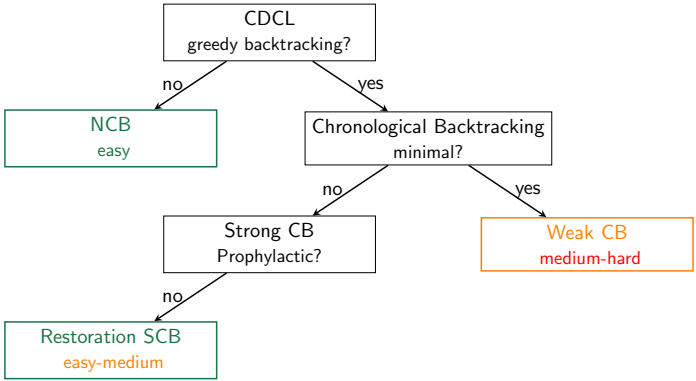
Backtracking Algorithm Implementation Complexity



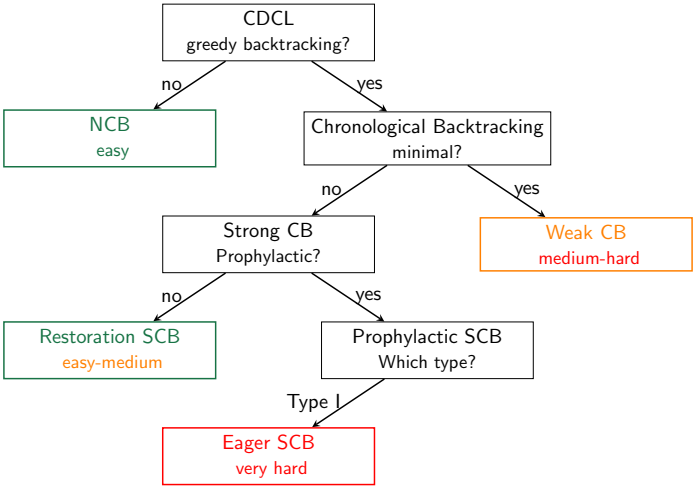
Backtracking Algorithm Implementation Complexity



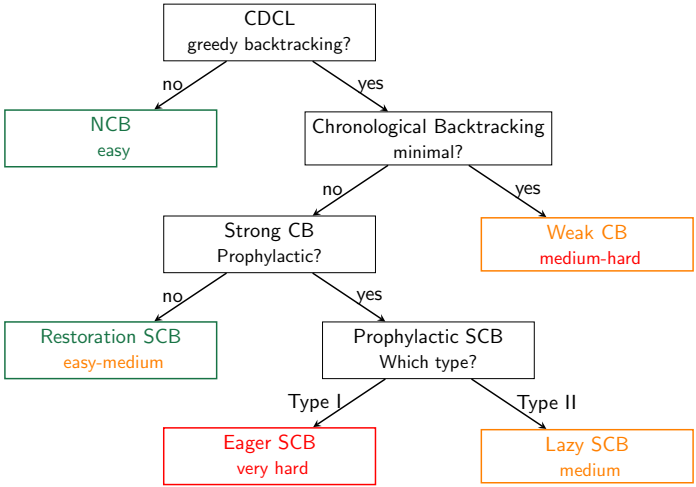
Backtracking Algorithm Implementation Complexity



Backtracking Algorithm Implementation Complexity



Backtracking Algorithm Implementation Complexity – Our Work



Conclusion

In this work, we

- formalized the dichotomy of chronological backtracking through invariants;
- introduced a new solution for missed lower implications;
- optimized conflict analysis with missed lower implications;
- implemented this new approach in a new SAT solver, NapSAT;
- implemented the method in CaDiCaL and Glucose to demonstrate its modularity;
- showed that the new method is competitive with the state-of-the-art in SAT solving.

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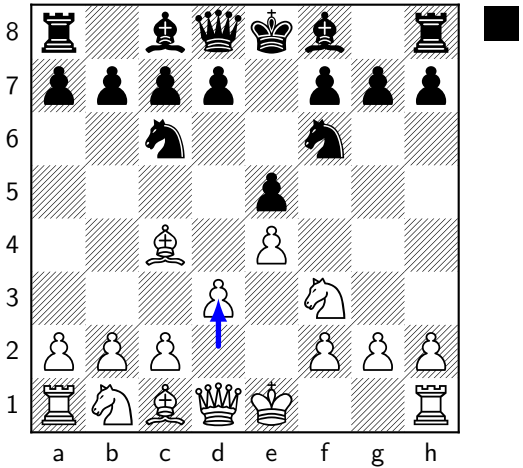
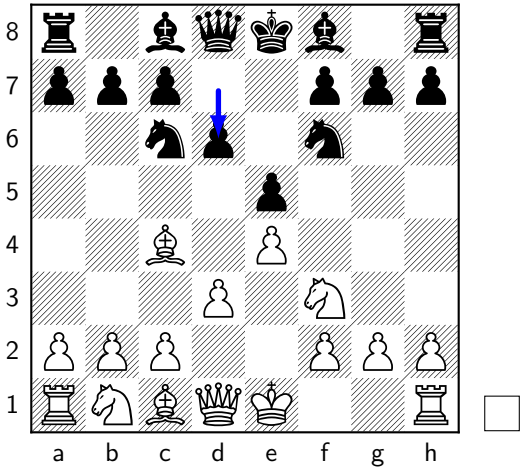
No Prophylaxis in Chess – Restoration

1 e4 e5 2 ♘f3 ♘c6 3 ♗c4 ♘f6 4 d3



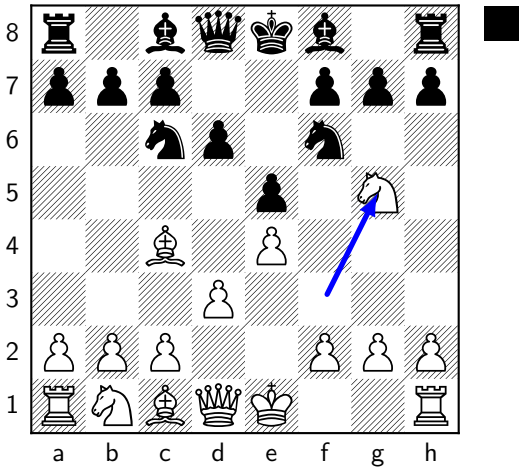
No Prophylaxis in Chess – Restoration

1 e4 e5 2 ♘f3 ♘c6 3 ♗c4 ♘f6 4 d3 d6



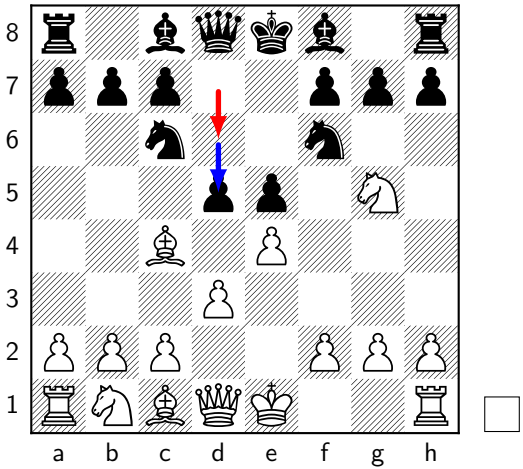
No Prophylaxis in Chess – Restoration

1 e4 e5 2 ♘f3 ♘c6 3 ♗c4 ♘f6 4 d3 d6 5 ♘g5



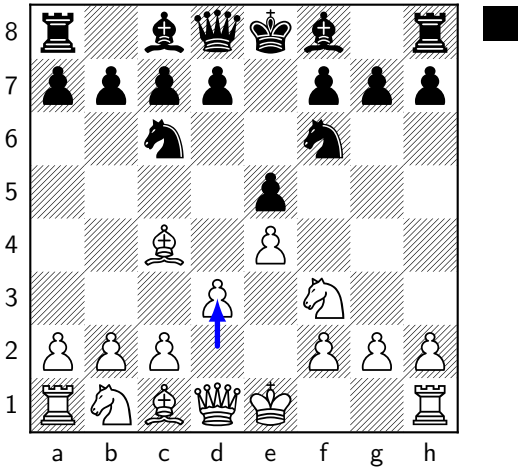
No Prophylaxis in Chess – Restoration

1 e4 e5 2 ♘f3 ♘c6 3 ♗c4 ♘f6 4 d3 d6 5 ♘g5 d5



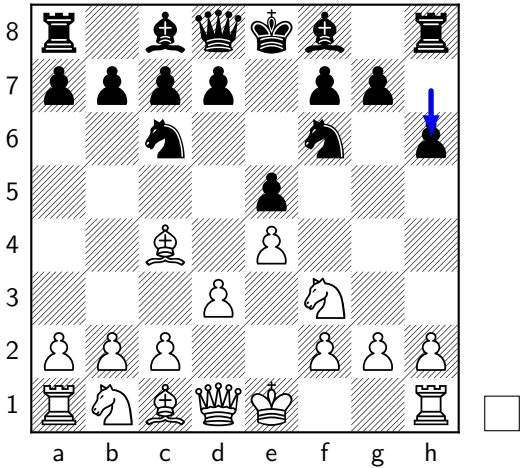
Type I Prophylaxis in Chess – Eager

1 e4 e5 2 ♘f3 ♘c6 3 ♗c4 ♘f6 4 d3



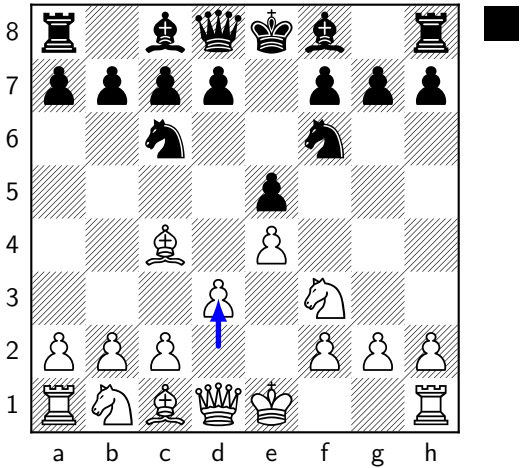
Type I Prophylaxis in Chess – Eager

1 e4 e5 2 ♘f3 ♘c6 3 ♗c4 ♘f6 4 d3 h6



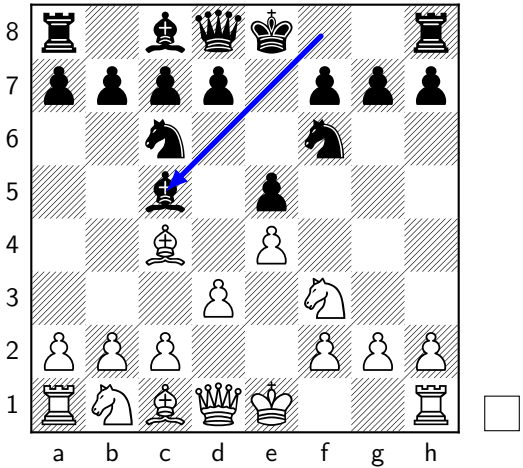
Type II Prophylaxis in Chess – Lazy

1 e4 e5 2 ♘f3 ♘c6 3 ♗c4 ♘f6 4 d3



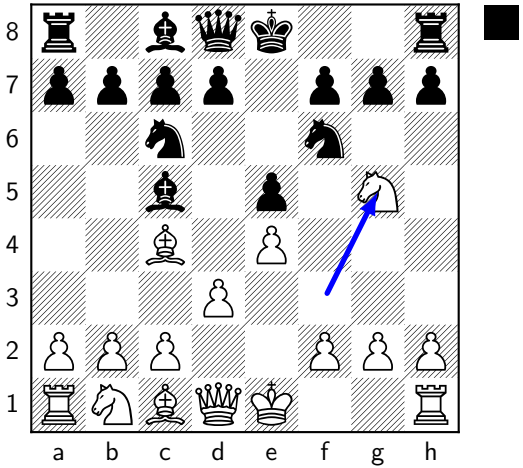
Type II Prophylaxis in Chess – Lazy

1 e4 e5 2 ♘f3 ♘c6 3 ♗c4 ♘f6 4 d3 ♗c5



Type II Prophylaxis in Chess – Lazy

1 e4 e5 2 ♘f3 ♘c6 3 ♗c4 ♘f6 4 d3 ♗c5 5 ♘g5



Type II Prophylaxis in Chess – Lazy

1 e4 e5 2 ♘f3 ♘c6 3 ♗c4 ♘f6 4 d3 ♗c5 5 ♘g5 O-O